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Executive Compensation: A Modern Primer†

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This article studies traditional and modern theories of executive compensation, bringing them together under a simple unifying framework accessible to the general-interest reader. We analyze assignment models of the level of pay, and static and dynamic moral-hazard models of incentives, and compare their predictions to empirical findings. We make two broad points. First, traditional theories find it difficult to explain the data, suggesting that compensation results from "rent extraction" by CEOs. However, more modern "shareholder value" theories, that arguably better capture the CEO setting, do deliver predictions consistent with observed practices, suggesting that these practices need not be inefficient. Second, seemingly innocuous features of the modeling setup, often made for tractability or convenience, can lead to significant differences in the model's implications and conclusions on the efficiency of observed practices. We close by highlighting apparent inefficiencies in executive compensation and additional directions for future research. (*JEL* G38, M12, M48, M52)

1. *Introduction*

There is considerable debate on executive compensation in both the public arena and academia. This debate spans several important topics in economics, such as contract theory, corporate finance, corporate governance, labor economics, and income inequality. One side is the "rent extraction" view, which claims that current compensation practices sharply contrast the predictions of traditional agency models. Thus, contracts are not chosen by boards to maximize shareholder value, but instead by the executives themselves to maximize their own rents. This perspective, espoused most prominently by Bebchuk and Fried (2004), has been taken very seriously by both scholars and policymakers, and led to major regulatory changes. In the United States, the Securities and Exchange Commission (SEC) mandated increased disclosure of compensation in 2006, and say-on-pay legislation was passed

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as part of the Dodd–Frank Act in 2010. In 2013, the European Union imposed caps on bankers' bonuses, the SEC mandated disclosure of the ratio of Chief Executive Officer (CEO) pay to median employee pay, and Switzerland held an ultimately unsuccessful referendum to limit CEO pay to twelve times the pay of the lowest worker.

The more modern "shareholder value" view reaches a different conclusion. While it acknowledges that elementary agency models are inconsistent with practice, it argues that such models do not capture the specifics of the CEO setting, since they were created as frameworks for the principal–agent problem in general. For example, CEOs have a very large effect on firm value compared to rank-and-file employees. Thus, in a competitive labor market, it may be optimal to pay high wages to attract talented CEOs, and implement high effort from them even though doing so requires paying a premium.[1] Newer models aim to capture the specifics of the CEO employment relationship, and can indeed generate predictions consistent with the data. Under this perspective, regulation will do more harm than good.

The "shareholder value" view broadens what is commonly referred to as the "optimal contracting" view, which typically focuses on the details of bilateral contracts. We use the term "shareholder value" for two main reasons. First, it emphasizes the need to take into account additional dimensions such as market forces and competitive equilibrium. Second, in reality boards are unlikely to choose the perfectly optimal contract, even if they are concerned with shareholder value, rather than rent extraction. One potential reason is a preference for simplicity, which may restrict them to piecewise linear contracts. The theoretically optimal contract is typically highly nonlinear and so never observed in reality; under a strict definition of optimal contracting, this view would be immediately rejected. A second is bounded rationality, which may lead to boards not being aware of certain (potentially nonobvious) performance measures that could theoretically improve the contract if included.

This article critically assesses the rent extraction versus shareholder value debate by analyzing newer models of executive compensation and evaluating the extent to which they can explain observed practices. In particular, while recent theories have used different frameworks and focused on different dimensions of the contracting problem, we present a tractable unifying model to bring together the conclusions of this large literature, starting with classic theories and then moving to modern ones. In section 2, we begin by analyzing the determinants of the level of pay, starting with neoclassical production models of the firm and then moving to modern assignment models. These assignment models yield empirical predictions for how CEO pay varies cross-sectionally between firms of different sizes, and over time as the size of the average firm in the economy changes.

Having determined the level of pay, we then move to incentives. In section 3, we consider a static moral-hazard model where the CEO takes an action that improves expected firm value, starting in section 3.1 with the risk-neutral case and moving to risk aversion in section 3.2. While this setting appears quite standard, we will show that seemingly innocuous features of the modeling setup, often made for convenience or tractability (e.g., the choice between additive or multiplicative utility and production functions, and binary or continuous actions) can lead to significant differences in the model's implications—and thus conclusions as to whether observed practices are consistent with theory. In particular, newer multiplicative models

[1] A simple model can justify high CEO pay simply by assuming a high level for the reservation utility, which is an exogenous parameter. Modern assignment models endogenize the reservation utility.

are able to explain some stylized facts (such as the relationship between incentives and firm size) that traditional additive models cannot. We also discuss various frameworks that researchers can use to yield tractable solutions to the contracting problem, and the appropriate empirical measure of incentives. Section 3.4 embeds the moral hazard model into a market equilibrium to generate additional empirical implications, and section 3.5 discusses the evidence. Section 3.6 considers the case of multiple signals. In contrast to the Holmstrom (1979) informativeness principle, in many firms the CEO's pay depends on industry shocks outside his control, which Bebchuk and Fried (2004) argue is strong evidence that contracting is suboptimal. We show that the theory does not unambiguously predict that industry shocks should be filtered out, due to other considerations in a CEO setting that are absent from Holmstrom (1979). Section 3.7 allows the CEO to affect the volatility as well as mean of firm value, by choosing the firm's risk. It discusses how options can encourage "good" risk taking, and debt-based compensation can deter "bad" risk shifting if the firm is levered.

Section 4 moves to a multi-period model. A dynamic setting poses several challenges absent from standard single-period models: contracts that are initially optimal may lose their incentive effect over time, the CEO can take myopic actions that boost short-term returns at the expense of long-run value, and he may undo the contract by private saving. In addition to these complications, a dynamic setting provides the principal with additional opportunities: she can provide incentives through the threat of termination, and base the CEO's pay on returns in previous, as well as current, periods.

Each section will compare the empirical predictions of the theories with the evidence. Broadly speaking, we will argue that many, but not all, features of observed contracts that are frequently criticized are actually consistent with efficiency, particularly when studying more modern theories. However, empirical correlations cannot be interpreted as definitive proof of the shareholder value view, given the difficulties in identifying causality. Section 5 highlights apparent inefficiencies in executive compensation, as well as open questions for future research. Section 6 concludes.

This article aims to differ from existing surveys of executive compensation. Core, Guay, and Larcker (2003) and Frydman and Jenter (2010) focus largely on the empirical evidence. Murphy (2013) provides a historical perspective and discusses the role of institutional constraints. Edmans and Gabaix (2009) focus exclusively on recent theories and use verbal descriptions, rather than a formal model. Our main contribution is to study both traditional and modern contracting theories, with a particular attention to the role of modeling choices, and combine their findings into a single unifying framework. As with any survey, we are forced to draw boundaries and so we focus on moral hazard, rather than adverse selection, as the former literature is more extensive. For learning models of CEO contracts, where either the CEO's general ability or his specific match quality with a firm is initially unknown, we refer the reader to Harris and Holmstrom (1982), Gibbons and Murphy (1992), Holmstrom (1999), Hermalin and Weisbach (1998, 2012), Taylor (2010, 2013), and Garrett and Pavan (2012).

2. *The Level of Pay*

Trends in the level of pay are perhaps the most commonly cited statistic in support of the rent-extraction view. The median CEO in the S&P 500 earned $9.6 million in 2011 (Murphy 2013), which is substantially higher than in other countries and represents a sixfold increase since 1980. In contrast, the pay of the average worker has risen much more

slowly. Figures from the Bureau of Labor Statistics show that CEO pay was 350 times that of the average worker in 2013, compared to 40 times in 1980, according to the Economic Policy Institute. Thus, the rapid increase in executive compensation may have contributed significantly to the recent rise in income inequality (Piketty and Saez 2003, Piketty 2014), and has potential political-economy implications. Bebchuk and Fried (2004) argue that this increase is a result of rent extraction by CEOs. Supporting this argument, Bebchuk, Cremers, and Peyer (2011) show that the fraction of CEO pay relative to total pay across the top-five executives is linked to lower firm value, profitability, and returns to acquisition announcements. We study the extent to which rises in pay over time can be explained by shareholder value models. In this section, we abstract from agency problems (which we later introduce in section 3) and study the pay required to attract the CEO to a firm.

2.1 *Talent as a Factor of Production*

One approach to determining the level of pay is to view the CEO as a factor of production separate from standard employees. Let the production function be

$$V = F(K, L, T),$$

where V is firm value and the factors of production are units of capital K, number of workers L (labor), and number of managers T. Each manager is paid a wage $w_T = \frac{\partial F}{\partial T}$. His pay is determined by the production function, and changes in pay result from shifts in technology. This is the perspective of most economic theories on the aggregate-production function and supply of talent (see Goldin and Katz 2009 and Acemoglu and Autor 2012 for recent surveys). In particular, labor economists use this perspective to compare the wages of, say, college graduates versus high-school dropouts.

The Lucas (1978) theory of the firm specializes the model to apply to the pay of a single CEO, rather than several managers. The variable T now refers to the CEO's level of human capital (i.e., his talent), rather than the number of managers. A CEO with talent T hires capital and labor[2] and maximizes

$$(1) \quad W_T = \max_{K,L} F(K, L, T) - w_L L - rK,$$

where w_L and r are the prices of labor and capital, and the surplus W_T is the CEO's pay. Consider the Cobb–Douglas production function

$$(2) \qquad V = T^{\alpha_T}\left(\frac{K}{\alpha_K}\right)^{\alpha_K}\left(\frac{L}{\alpha_L}\right)^{\alpha_L},$$

where α_T, α_K, and α_L represent the shares of output that go to the CEO, capital, and labor, respectively, under perfect competition. We assume $\alpha_T + \alpha_K + \alpha_L = 1$ (constant returns to scale). The first-order condition of (1) with respect to K is $\alpha_K \frac{V}{K} = r$ i.e., $\frac{K}{\alpha_K} = \frac{V}{r}$. Optimizing over labor likewise yields $\frac{L}{\alpha_L} = \frac{V}{w_L}$. Substituting into the production function (2) gives

$$V = T^{\alpha_T}\left(\frac{V}{r}\right)^{\alpha_K}\left(\frac{V}{w_L}\right)^{\alpha_L}$$

$$= \frac{1}{r^{\alpha_K} w_L^{\alpha_L}} T^{\alpha_T} V^{1-\alpha_T}.$$

Solving for V, we have

$$(3) \qquad V = (r^{\alpha_K} w^{\alpha_L})^{-1/\alpha_T} T;$$

$$K = \frac{\alpha_K V}{r}; L = \frac{\alpha_L V}{w_L}.$$

[2] An alternative formulation is for capital to hire the CEO and labor. In competitive markets, the identity of the principal is immaterial. Here, we follow the Lucas (1978) formulation for ease of exposition.

From (3), a more talented CEO runs a larger firm, in part because he hires more capital and labor: V, K, L are all linear in T. His pay is given by

$$(4) \qquad W_T = V - rK - w_L L = \alpha_T V.$$

The model generates the qualitative prediction that CEO pay, W_T, is increasing in firm size, because a larger firm generates more surplus. It also generates the quantitative prediction that his pay scales linearly with firm size. Various empirical studies confirm the qualitative prediction that CEO pay is increasing in firm size[3]: Baker, Jensen, and Murphy (1988, p. 609) call this relationship "the best documented empirical regularity regarding levels of executive compensation." However, the quantitative prediction that pay is linear in firm size is contradicted by the data. The above papers find that CEO pay increases as a power function of firm size $W_T \sim S^{\kappa}$, where a typical elasticity is $\kappa \simeq 1/3$. Hence, the Lucas model needs to be refined. This is what assignment models do, to which we now turn.

2.2 *Assignment Models*

Gabaix and Landier (2008) present a tractable market-equilibrium model of CEO pay. A continuum of firms and potential CEOs are matched together. Firm $n \in [0, N]$ has a "baseline" size $S(n)$ and CEO $m \in [0, N]$ has talent $T(m)$. Low n denotes a larger firm and low m a more talented CEO: $S'(n) < 0$, $T'(m) < 0$. The value $n(m)$ can be thought of as the rank of the firm (CEO), or a number proportional to it, such as its quantile of rank.

We consider the problem faced by one particular firm. At $t = 0$, it hires a CEO of talent $T(m)$ for one period. The CEO's talent increases firm value according to

$$(5) \qquad V = S(n) + CS(n)^{\gamma} T(m),$$

where C parametrizes the productivity of talent and γ the elasticity of talent with respect to firm size. If $\gamma =(<)\ 1$, the model exhibits constant (decreasing) returns to scale.

We now determine equilibrium wages, which requires us to allocate one CEO to each firm. Let $w(m)$ denote the equilibrium wage of a CEO with index m. Firm n, taking the market wage of CEOs as given, selects CEO m to maximize its value net of wages:

$$\max_m CS(n)^{\gamma} T(m) - w(m).$$

The competitive equilibrium involves positive assortative matching, i.e., $m = n$, and so $w'(n) = CS(n)^{\gamma} T'(n)$. Let $\underline{w}_N$ denote the reservation wage of the least talented CEO (who is matched to firm $n = N$). Hence we obtain the classic assignment equation (Sattinger 1993), also derived by Terviö (2008) in the context of CEOs:

$$(6) \qquad w(n) = -\int_n^N CS(u)^{\gamma} T'(u)\, du + \underline{w}_N.$$

Specific functional forms are required to proceed further. We assume a Pareto firm-size distribution with exponent $1/\alpha$: $S(n) = An^{-\alpha}$. Using results from extreme value theory, Gabaix and Landier (2008) use the following asymptotic value for the spacings of the talent distribution: $T'(n) = -Bn^{\beta-1}$. These functional forms give the wage in closed form, taking the limit as $n/N \to 0$:

$$(7) \qquad w(n) = \int_n^N A^{\gamma} BC\, u^{-\alpha\gamma+\beta-1}\, du + \underline{w}_N$$

$$= \frac{A^{\gamma} BC}{\alpha\gamma - \beta} \left[n^{-(\alpha\gamma-\beta)} - N^{-(\alpha\gamma-\beta)} \right]$$

$$+ \underline{w}_N \sim \frac{A^{\gamma} BC}{\alpha\gamma - \beta} n^{-(\alpha\gamma-\beta)}.$$

[3] See, e.g., Roberts (1956), Cosh (1975), Baker, Jensen, and Murphy (1988), Barro and Barro (1990), Kostiuk (1990), Rosen (1992), Joskow, Rose, and Shepard (1993), Rose and Shepard (1997), and Frydman and Saks (2010).

To interpret equation (7), we consider a reference firm, for instance the median firm in the universe of the top 500 firms. Denote its index n_*, and its size $S(n_*) = An_*^{-\alpha}$. Using $S(n) = An^{-\alpha}$, we derive:

$$\begin{aligned} w(n) &= \frac{A^\gamma BC}{\alpha\gamma - \beta} n^{-(\alpha\gamma-\beta)} \\ &= \frac{A^\gamma BC}{\alpha\gamma - \beta} \left(\left(A^{1/\alpha} S(n)^{-1/\alpha}\right)\right)^{-(\alpha\gamma-\beta)} \\ &= \frac{A^{\beta/\alpha} BC}{\alpha\gamma - \beta} S(n)^{\gamma-\beta/\alpha} \\ &= \frac{\left(S(n_*) n_*^\alpha\right)^{\beta/\alpha} BC}{\alpha\gamma - \beta} S(n)^{\gamma-\beta/\alpha} \\ &= \frac{n_*^\beta BC}{\alpha\gamma - \beta} S(n_*)^{\beta/\alpha} S(n)^{\gamma-\beta/\alpha}. \end{aligned}$$

Finally, we obtain CEO pay in closed form:

$$(8) \quad w(n) = D(n_*) S(n_*)^{\beta/\alpha} S(n)^{\gamma-\beta/\alpha},$$

where $D(n_*) = -Cn_* T'(n_*)/(\alpha\gamma - \beta)$ is a constant independent of firm size. Similar to Lucas (1978), equation (8) yields the qualitative cross-sectional prediction that CEO pay is increasing in firm size. However, the intuition is different: here the prediction arises because large firms hire the most talented CEOs, who command the highest wages. Moreover, equation (8) yields a different quantitative prediction. It predicts a pay–firm size elasticity of $\rho = \gamma - \beta/\alpha$. Gabaix and Landier (2008) calibrate using $\alpha = 1$ (a Zipf's law, as in Axtell 2001 and Gabaix 1999) and $\gamma = 1$ (constant returns to scale). Since there is no clear a priori value for β, they set $\beta = 2/3$ to yield the empirical pay–size elasticity of $\rho = 1/3$, which contrasts Lucas's (1978) prediction of $\rho = 1$. Baranchuk, MacDonald, and Yang (2011) extend the model to endogenize firm size and show that the pay–size relationship is stronger when industry conditions are favorable, as talented CEOs are not only paid a greater premium, but also optimally grow their firms to a larger size.

In addition, equation (8) shows that pay increases with the size of the average firm in the economy $S(n_*)$. Since a CEO's talent can be applied to the entire firm, when firms are larger, the dollar benefits from a more talented CEO are higher, so there is more competition for talent. This is a similar "superstars" effect to Rosen (1992). Moreover, the model's closed form solutions yield quantitative predictions. Average firm size increased sixfold between 1980 and 2011. When both $S(n_*)$ and $S(n)$ rise by a factor of 6, CEO pay should rise by a factor of $6 \times \left[\beta/\alpha + (\gamma - \beta/\alpha)\right] = 6\gamma = 6$, as has been the case. The relevant benchmark against which to compare the level of CEO pay is not the pay of the average worker, or pay of CEOs in the past, but the CEO's current contribution to the firm. This in turn depends on variables such as firm size and talent; while the latter is difficult to measure, the pay of the average worker is unlikely to be a determinant. Thus, assignment models suggest that regulation to mandate disclosure of the ratio of CEO pay to median employee pay may not be useful, as median employee pay is not the relevant benchmark.

However, the empirical evidence is not unambiguously in favor of assignment models. Nagel (2010) raises sample selection concerns and suggests alternative methodologies, but Gabaix, Landier, and Sauvagnat (2014) conclude that the results are robust to these changes. While Gabaix and Landier (2008) can fully explain the growth in CEO pay from 1980 to the present, Frydman and Saks (2010) find that median CEO pay was relatively constant between the 1940s and early 1970s, despite firm size increasing over this period. Gabaix and Landier (2008) discuss potential explanations for this apparent discrepancy. One is that the supply of talent

greatly increased, which creates a downward pressure on CEO wages; quantifying the impact of increased supply on wages would be a useful direction for future research. Another is that, in the early period, CEOs tended to be internally promoted rather than externally hired, similiar to the Japanese CEO market today.

In addition, observing that both firm size and CEO pay have trended upwards since 1980 does not imply causality. Even if causal, the positive correlation between pay and firm size cannot be interpreted as definitive evidence for assignment models, since it is also potentially explainable by an as-yet unwritten rent-extraction model. For example, large firms may have more resources, allowing the CEO to extract more salary. Alternatively, Dicks (2012) shows that the correlation can arise if poor governance causes a small fraction of firms to overpay for talent, which then forces all others to overpay as well, in order to remain competitive. This channel is also predicted by Gabaix and Landier (2008); see Bereskin and Cicero (2013) for supportive evidence. While these alternative explanations would generate the qualitative prediction that pay is positively correlated with firm size and average firm size, it is not yet clear whether they can generate empirically consistent quantitative predictions.

Another concern is that assignment models predict a reassignment of CEOs as relative firm size changes. While external poaching of CEOs has increased in recent years, it is still relatively rare: Cremers and Grinstein (2014) find that 63 percent of new CEOs are insiders. Similarly, it does not appear to be the case that large changes in firm size (e.g., a firm undertaking a large acquisition) are accompanied by changes in the acquiring CEO for non-disciplinary reasons. This low mobility can be generated by a simple extension of assignment models to incorporate frictions, such as a cost of firing the CEO or firm-specific human capital.[4] Writing and calibrating such a model would be valuable.

Rather than using firm size as a proxy for talent, other authors have attempted to measure talent directly. Chang, Dasgupta, and Hilary (2010) infer talent from the market reaction to CEO departure, which they find is positively related to pay. To address concerns that the market reaction captures perception, rather than true ability, they show that it is negatively correlated to predeparture performance. Falato, Li, and Milbourn (2015) directly measure ability using a CEO's reputational, career, and educational credentials, which they corroborate by showing a positive association with future firm performance. They find that such credentials are positively related to pay, consistent with talent-based models.

2.3 *Alternative Explanations for High CEO Pay*

Moving to other explanations for high CEO pay, section 3 discusses how agency problems may lead to the CEO being paid a premium for the disutility of effort and the risk imposed by incentives. Other papers point to the changing nature of the employment relationship. Hermalin (2005) argues that tighter corporate governance increases both the level of effort that the CEO must exert and the risk of dismissal, and so managers demand greater pay as a compensating differential. Indeed, Peters and Wagner (2014) show that CEO turnover risk is significantly positively associated with pay, but reject an entrenchment model in which powerful CEOs enjoy both lower turnover risk and high pay. Their

[4] In addition, Cremers and Grinstein (2014) find the relationship between pay and firm size differs little across industries, according to the proportion of outsider CEOs in the industry. They interpret this proportion as a potential measure of competition for CEO talent. However, this proportion may reflect small frictions that cause a firm to choose an insider CEO on the margin, but are not large enough to meaningfully affect equilibrium pay.

identification strategy focuses on industry volatility, which (after controls) is unlikely to affect CEO pay other than through turnover risk. In Garicano and Rossi-Hansberg (2006), CEOs specialize in knowledge acquisition and problem solving, leaving routine production tasks to lower-level employees. Recent increases in communication technologies (e.g., e-mail) allow the CEO to specialize more in skilled tasks, thus increasing his pay. Frydman (2014) and Murphy and Zabojnik (2007) argue that the increasing importance of transferable, rather than firm-specific, human capital increases pay through expanding CEOs' outside options.

Moreover, despite the significant relationship between pay and factors such as firm size, risk, and (as we will discuss in section 3) performance, Graham, Li, and Qiu (2012) find that a large component is explained by manager fixed effects. Inclusion of these fixed effects changes the coefficient estimates on other determinants; for example, the firm-size coefficient falls by 40 percent. These fixed effects could be a proxy for talent (consistent with their effect on the firm size coefficient), for the manager's ability to extract rent, or other factors such as managerial preferences, risk aversion, or personality. Thus, a significant proportion in the variance of firm pay remains unexplained.

The above explanations—talent, agency, and the changing nature of the CEO's job—are part of the shareholder value view. To test this view more broadly, Cronqvist and Fahlenbrach (2013) study the effect on compensation of firms transitioning from public to private ownership. Since the private equity sponsor's concentrated stake gives it the incentives and control rights to set pay optimally, compensation in private firms should be closer to the efficient benchmark. Salary and bonuses actually rise upon going private, inconsistent with the notion that CEOs of public firms are overpaid. They issue the caveat that their results are based on twenty leveraged buyouts. Due to data limitations, their inferences are based on differences in means without controls, but they show that these changes do not occur in control firms that experienced similar increases in leverage—i.e., it is likely concentrated ownership, rather than leverage, that explains the results. More generally, Kaplan (2012) reports that over the past three decades, executive pay in closely held firms has outpaced that in public companies.

In addition to the value-maximization and rent-extraction views, a third perspective is that institutional constraints or practices may have contributed to the rise of pay. Murphy (2013) discusses the role of tax policy, accounting rules, and disclosure requirements: for example, the Clinton Administration's $1 million salary cap led to many firms increasing salary to $1 million. Shue and Townsend (forthcoming a) note that firms tend to grant the same number of options each year. Thus, when stock prices rise, the value of options increases which, together with downward rigidity in salaries and bonuses, led to overall pay levels rising in the 1990s and early 2000s. While this friction is indeed significant in the short run, its effect on long-run outcomes is less clear—similar to economics more broadly, where pricing frictions (e.g., menu costs) are important in the short run but less so in the long run.

3. *Static Incentives*

We now turn from determining the level of pay to the CEO's incentives. This section considers a single-period moral hazard model, which we extend to multiple periods in section 4. This setting has been widely covered in textbooks (e.g., Bolton and Dewatripont 2005, Tirole 2006) and earlier surveys (Prendergast 1999), but typically with additive production functions and preferences, and often a binary effort level. We will show that multiplicative specifications,

which may be particularly relevant for a CEO setting, lead to quite different conclusions for the best empirical measure of incentives and how incentives should vary cross-sectionally between firms. We will also show that the use of a continuous versus binary action space, as well as the specification of noise before versus after the action, also lead to significant differences in the model's results.

We start with a standard principal–agent problem applied to an executive compensation setting. The principal (board of directors on behalf of shareholders) hires an agent (CEO) to run the firm. The production function is given by $V(a, S, \varepsilon)$, which is increasing in a and S. We specialize this to

$$V = S + b(S)\,a + \varepsilon. \tag{9}$$

We consider an all-equity firm for simplicity and discuss leverage in section 3.7. The variable $a \in [0, \infty)$ is an action taken by the agent that improves expected firm value but is personally costly. Examples include effort (low a represents shirking), project choice (low a involves selecting value-destructive projects that maximize private benefits), or rent extraction (low a reflects cash flow diversion.) We typically refer to a as "effort" for brevity. The variable ε is mean-zero noise, with interval support on $(\underline{\varepsilon}, \overline{\varepsilon})$, where the bounds may be infinite.[5] Shortly after the agent takes his action, noise is realized, and then final firm value V is realized. Firm value is observable and contractible, but neither effort nor noise is individually observable.

The function $b(S)$ measures the effect of effort on firm value for a firm of size S. One possibility is $b(S) = b$, which yields $V(a) = S + ba + \varepsilon$: an additive production function where the effect of effort on firm value is independent of initial firm size. This specification is appropriate for a perk-consumption decision, if the amount of perks that can be consumed is independent of firm size. For example, buying a \$10 million corporate jet reduces firm value by \$10 million, regardless of S. Another is $b(S) = bS$, which yields $V(a) = S(1 + ba) + \varepsilon$: a multiplicative production function where the effect of firm value is linear in firm size. Many CEO actions can be "rolled out" across the entire firm and thus have a greater effect in a larger company, such as a change in strategy or a program to improve production efficiency. A multiplicative specification is also appropriate for a rent-extraction setting, if there are greater resources to extract in a larger firm.[6]

The agent is paid a wage $c(V)$ contingent upon firm value. (Note that c refers to actual pay, in contrast to w which refers to the expected wage). We always assume limited liability on the principal ($c(V) \leq V$): she cannot pay out more than total firm value. In some versions of the model, we will also assume limited liability on the agent ($c(V) \geq 0$). He has reservation utility of $w \geq 0$ and his objective function is given by

$$E[U] = E[u(v(c) - g(a))]. \tag{10}$$

The function g represents the cost of effort, which is increasing and weakly convex. Further, u is the utility function and v is the felicity[7] function, which denotes the agent's utility from cash; both are increasing and weakly concave. The functions g, u, and v are all twice continuously differentiable. The objective function (10) contains functions for both utility and felicity to maximize

[5] For simplicity, we assume that S is sufficiently large, or the probability of low ε is sufficiently small, that V is nonnegative almost surely, so we do not need to complicate the model with nonnegativity constraints.

[6] See Bennedsen, Perez-Gonzalez, and Wolfenzon (2010) for empirical evidence that CEOs have the same percentage effect on firm value regardless of firm size.

[7] We note that the term "felicity" is typically used to denote one-period utility in an intertemporal model. We use it in a nonstandard manner here to distinguish it from the utility function u.

generality. One common assumption is $v(c) = c$ so that $E[U] = E[u(c - g(a))]$, in which case the cost of effort is pecuniary, i.e., can be expressed as a subtraction to cash pay. This is appropriate if effort involves a financial expenditure or the opportunity cost of foregoing an alternative income-generating activity. Another is $u(x) = x$, which yields $E[v(c) - g(a)]$, where the cost of effort is separable from the benefits of cash. This specification is reasonable if effort involves disutility, or foregoing leisure or private benefits.

Both of the above specifications represent additive preferences. Effort of a reduces the agent's utility by $g(a)$ in utils (dollars) in the first (second) specification. A third specification is $v(c) = \ln c$, in which case (10) becomes $E\left[u\left(\ln\left(ce^{-g(a)}\right)\right)\right]$. This specification corresponds to multiplicative preferences, where the cost of effort is increasing in c. Here, private benefits are a normal good: the utility they provide is increasing in consumption, consistent with the treatment of most goods and services in consumer theory. This specification is also plausible under the literal interpretation of effort as foregoing leisure: a day of vacation is more valuable to a richer CEO, as he has more wealth to enjoy during it. Thus, the CEO's expenditure on leisure and private benefits rises in proportion to his wealth. Multiplicative preferences are also commonly used in macroeconomic models (e.g., Cooley and Prescott 1995) to generate realistic income effects. In particular, they are necessary for labor supply to be constant over time as the hourly wage rises.[8]

The principal is assumed to be risk neutral, since shareholders are typically well-diversified. Her program is given by:

$$\max_{c(\cdot),a} E[V(a) - c(V(a))] \quad \text{s.t.} \tag{11}$$

$$E[u(v(c(V(a))) - g(a))] \geq w, \tag{12}$$

$$a \in \arg\max_{\hat{a}} E[u(v(c(V(\hat{a}))) - g(\hat{a}))]. \tag{13}$$

She chooses the effort level a and contract $c(V)$[9] to maximize (11), expected firm value minus the expected wage, subject to the agent's individual rationality or participation constraint (IR, (12)) and incentive compatibility constraint (IC, (13)).

We begin with a first-best benchmark, which leads to a simple optimal contract that is the same across all firms and thus does not have the potential to explain observed contracts. We then explore two departures from the first-best that generate a meaningful contract that does yield empirical predictions. The first is limited liability, which only leads to small variations in the optimal contract across firms. The second is risk aversion (section 3.3), which leads to much richer implications. Section 3.4 embeds the contracting problem in a market equilibrium to generate additional empirical implications. We compare all implications to the data in section 3.5.

Under the first-best, effort is observable. Let a^* be the effort level that the principal wishes to implement. She can simply direct the agent to exert effort a^*, and so we can ignore the IC (13). It is easy to show that the

[8] When the hourly wage rises, working becomes preferable to leisure (the substitution effect). With multiplicative preferences, the rise in the wage increases the agent's labor endowment income and thus demand for leisure (the income effect), which exactly offsets the substitution effect. With additive preferences, there is no income effect, and so leisure falls to zero as the wage increases.

[9] Here, we focus on deterministic contracts, so that there is a one-to-one mapping between firm value V and compensation c. An even more general model allows for stochastic contracts, where firm value of V leads to a random amount c. Gjesdal (1982), Arnott and Stiglitz (1988), and Edmans and Gabaix (2011b) derive sufficient conditions for random contracts to be suboptimal, allowing the focus on deterministic contracts.

agent is given a constant wage $c(V) = \bar{c}$, as this leads to efficient risk sharing. The IR (12) yields $\bar{c} \geq w + g(a^*)$. This will bind in the optimal contract, and so the principal maximizes

$$E[V(a^*)] - g(a^*) - w. \tag{14}$$

This defines the first-best effort level as

$$g'(a^*_{FB}) = b(S). \tag{15}$$

The principal trades off the marginal increase in firm value from effort, $b(S)$, with the agent's marginal cost, $g'(a^*_{FB})$. Thus, a^*_{FB} maximizes total surplus. In turn, a^*_{FB} is decreasing in the convexity of the cost of effort. It is also increasing in firm size S if $b(S)$ is increasing in S, since effort then has a greater dollar effect in a larger firm.

We now turn to a setting in which effort is unobservable and the IC (13) must be imposed. We first assume a risk-neutral agent, before moving to risk aversion.

3.1 *Risk-Neutral Agent*

We first consider risk neutrality and additive preferences. We have $u(x) = x$ and $v(c) = c$ so the participation and incentive constraints (12) and (13) specialize to

$$E[c(V)] - g(a) \geq w \tag{16}$$

and

$$a \in \arg\max_{\hat{a}} E[c(V)] - g(\hat{a}). \tag{17}$$

Grossman and Hart (1983) show that the contracting problem can be solved in two stages, which correspond to the principal's two choice variables. She first chooses the contract $c(V)$ that implements a given action a^* at least cost, and then the optimal a^* taking into account the cost of the contract $c(V)$ needed to implement each action a^*. Starting with the first stage, the first-order condition of the agent's effort choice (17) is given by

$$E[c'(V)b(S)] = g'(a^*). \tag{18}$$

Rogerson (1985), Jewitt (1988), and Carroll (2012) give conditions under which the first-order condition is sufficient, and so the IC (17) can be replaced by the first-order condition (18), which greatly simplifies the problem. Throughout this paper, we assume that these conditions are satisfied, so that the first-order approach is valid.

Given risk neutrality and unlimited liability, there is no loss of generality in focusing on a linear contract of the form $c(V) = \phi + \theta V$, where ϕ is the fixed wage and θ is the agent's percentage stake in firm value. Then, using (18), in order to implement effort of a^*, the CEO's incentives are given by

$$\theta b(S) = g'(a^*). \tag{19}$$

Empiricists typically measure the CEO's incentives to improve firm value, i.e., to exert effort a^*. Equation (19) shows how the optimal measure of incentives depends on how we specify the production function. When it is additive ($b(S) = b$), then to implement a given effort level a^*, the firm must set correctly the incentive measure θ, the agent's percentage stake in firm value V. This measure corresponds to the dollar change in pay for a one dollar change in firm value ($–$ incentives) and is used by Demsetz and Lehn (1985) and Jensen and Murphy (1990), among others.

Hall and Liebman (1998) argue that most CEO actions have a multiplicative effect on firm value. With a multiplicative production function ($b(S) = bS$), we have $\theta bS = g'(a^*)$, and so the relevant incentive measure is θS, the CEO's dollar equity stake. This measure

corresponds to the dollar change in pay for a one percentage point change in firm value (\$–% incentives).

Thus, while it is common to assume an additive production function for simplicity, researchers should think carefully about how to specify these functions as this choice has important implications for the relevant measure of incentives—a point first noted by Baker and Hall (2004). Moreover, if CEO effort has a multiplicative effect on firm value, then CEO incentives are a quantitatively much more important issue than his level of pay, even though the latter receives much greater attention in the media. While a \$9.6 million salary is substantial compared to average worker pay, relative to a \$10 billion firm it constitutes 0.1 percent of firm value. In contrast, if incentives are insufficient to induce the CEO to implement a major restructuring or reject a bad acquisition, the losses to shareholders could run into several percentage points.

Before moving to the second stage of Grossman and Hart (1983), we demonstrate the effect of multiplicative preferences, as studied by Edmans, Gabaix, and Landier (2009). In the general objective function (10), this corresponds to $u(x) = e^x$ and $v(c) = \ln c$, which yields

$$U = E\left[c e^{-g(a)}\right].$$

We normalize $a^* = 0$, and so the $t = 0$ stock price (net of CEO pay) is S.[10] In (9) we have $b(S) = bS$, i.e., a multiplicative production function, so that firm value at $t = 1$ is given by

$$V(a) = S(1 + ba) + \varepsilon.$$

[10] For simplicity, we assume that initial firm size S is net of the expected wage w.

The IR is given by $E[c \mid a = a^*] = w$, which yields

$$w = [c|a = a^*] = \phi + \theta E[V|a = a^*] = \phi + \theta S.$$

If the CEO exerts effort a, his utility is

$$\begin{aligned} E[U(a)] &= E\left[c(a)e^{-g(a)}\right] \\ &= (\phi + \theta EV(a))e^{-g(a)} \\ &= (\phi + \theta S(1 + ba))e^{-g(a)} \\ &= (w + \theta Sba)e^{-g(a)} \\ &= w\left(1 + \frac{\theta Sb}{w}a\right)e^{-g(a)} \\ &= we^{\ln\left(1+\frac{\theta Sb}{w}a\right)-g(a)}. \end{aligned}$$

The IC is $a^* \in \arg\max_a E[U(a)]$. At $a^* = 0$, this yields $E[U'(0)] = 0$, i.e.,

$$\frac{\theta S}{w} = \frac{g'(a^*)}{b}. \tag{20}$$

Thus, to implement a given effort level a^*, the firm must set correctly the incentive measure $\frac{\theta S}{w}$, i.e., the CEO's dollar equity stake scaled by his annual pay, or alternatively the fraction of total pay w that is in equity. It corresponds to the percentage change in pay for a 1 percentage point change in firm value (%–% incentives, i.e., the elasticity of pay to firm value), as used by Murphy (1985), Gibbons and Murphy (1992), and Rosen (1992).

Using θ^I, θ^{II}, and θ^{III}, respectively, to denote %–%, \$–\$, and \$–% incentives, we have:

$$\theta^I = \frac{\partial c}{\partial r}\frac{1}{w} = \frac{\Delta \ln \text{Pay}}{\Delta \ln \text{Firm Value}}; \tag{21}$$

$$\theta^{II} = \frac{\partial c}{\partial r}\frac{1}{S} = \frac{\Delta\$\text{Pay}}{\Delta\$\text{Firm Value}}; \tag{22}$$

$$\theta^{III} = \frac{\partial c}{\partial r} = \frac{\Delta\$\text{Pay}}{\Delta \ln\text{Firm Value}}, \tag{23}$$

where $r = V/S - 1$ is the firm's stock market return. In our one-period model, incentives arise from new grants of stock and options, plus changes in cash pay (salary and bonuses). Thus, these incentive measures are referred to as "pay–performance sensitivity." In reality, CEOs are in office for multiple periods, and the vast majority of incentives stem from changes in the value of previously granted stock and options, which swamp changes in cash pay (Jensen and Murphy 1990, Hall and Liebman 1998). Replacing flow compensation c in the numerator of expressions (21) to (23) with the CEO's wealth W yields analogous expressions for "wealth–performance sensitivity," the change in the CEO's entire wealth (including previously granted stock and options) for a change in firm performance:

$$\Theta^{I} = \frac{\partial W}{\partial r}\frac{1}{w} = \frac{\Delta \ln\text{Wealth}}{\Delta \ln\text{Firm Value}}; \tag{24}$$

$$\Theta^{II} = \frac{\partial W}{\partial r}\frac{1}{S} = \frac{\Delta\$\text{Wealth}}{\Delta\$\text{Firm Value}}; \tag{25}$$

$$\Theta^{III} = \frac{\partial W}{\partial r} = \frac{\Delta\$\text{Wealth}}{\Delta \ln\text{Firm Value}}. \tag{26}$$

For example, $\Theta^{I} = \frac{\partial W}{\partial r}\frac{1}{w}$ is the percentage change in wealth for a 1 percentage point change in the stock return, scaled by annual pay, which Edmans, Gabaix, and Landier (2009) call "scaled wealth–performance sensitivity." Empirical studies should consider wealth–performance sensitivities, rather than pay–performance sensitivities, since the latter only capture a small part of the CEO's incentives. Indeed, Core, Guay, and Thomas (2005) overturn Bebchuk and Fried's (2004) conclusion that CEOs have weak incentives when studying wealth– rather than pay–performance sensitivities. Section 3.4 predicts how the three incentive measures scale with firm size and section 3.5 will test these predictions. These tests will shed light on whether utility and production functions are additive or multiplicative, and thus the optimal measure of incentives.

We now solve for the second stage of Grossman and Hart (1983), i.e., the optimal effort level, returning to the case of additive preferences. If the agent exhibits unlimited liability, the principal can always adjust fixed pay ϕ so that the participation constraint (16) binds. Thus, his expected pay is $E[c(V)] = w + g(a^*)$, just as in the first-best, and so the principal's objective function remains (14). As a result, she implements the first-best effort level, defined by (15). Using (15) and (18), the optimal contract satisfies

$$E[c'(V)b(S)] = b(S). \tag{27}$$

With a linear contract, this yields $\theta = 1$ and so the optimal contract is given by

$$c(V) = \phi + V, \quad \text{where} \tag{28}$$

$$\phi = w + g(a^*) - S - b(S)\,a^*. \tag{29}$$

The principal effectively "sells" the firm V to the agent for an up-front fee of $-\phi$, chosen so that the participation constraint (16) binds. Since the agent benefits one-for-one from any increase in firm value, he fully internalizes the benefits of effort and the first-best effort level a^*_{FB} is achieved. The level of incentives is "one size fits all": regardless of the cost or utility function, we have $\theta = 1$.

In the above framework, the effort level a^*_{FB} is chosen endogenously and so the principal implements whatever effort level is implied by $\theta = 1$. One simple way to

obtain meaningful contracts that do differ across firms is to consider a binary effort decision, $a \in \{\underline{a}, \overline{a}\}$, where the principal implements $\overline{a}$, as in Holmstrom and Tirole (1997); Edmans, Gabaix, and Landier (2009); Biais et al. (2010); and the textbook by Tirole (2006). A similar specification is a continuous but bounded action space, $a \in [\underline{a}, \overline{a}]$, where again the principal wishes to implement $\overline{a}$. The upper bound reflects the fact that there may be a limit to the number of actions that a CEO can take to increase firm value. The high effort level $\overline{a}$ represents full productive efficiency, rather than working twenty-four hours a day. In a cash flow diversion model, full productive efficiency corresponds to zero stealing; in a project-selection model, it corresponds to taking all positive net present value (NPV) projects while rejecting negative NPV ones; in an effort model, it corresponds to the CEO not deliberately refraining from an action that will improve firm value because he prefers to shirk. Then, from equations (19) and (20), the optimal incentive level is $\theta b(S) = g'(\overline{a})$ if utility is additive and $\frac{\theta b(S)}{w} = g'(\overline{a})$ if utility is multiplicative.[11] Thus, the optimal level of incentives (\$–\$, \$–%, or %–%, depending on the model specification) is increasing in the cost of effort $g'(\overline{a})$. Incentives are higher in firms with greater agency problems, rather than one size fits all.

The first-best is still achieved in the fixed-action setting. In reality, the first-best cannot be achieved for two reasons. First, the agent may be subject to limited liability ($c(V) \geq 0$). Under contract (28), the agent will receive a negative payoff if V is sufficiently low, violating limited liability. Put differently, the agent may not have enough cash to buy the firm. Second, he may be risk averse and demand a premium for bearing the risk associated with firm value V. We explore these two departures in turn and show that they both lead to nontrivial contracts.

[11] When a is a boundary action, the IC becomes an inequality and a continuum of contracts will implement $a = \overline{a}$. We choose the contract that involves the minimum amount of incentives, as this is optimal for any nonzero level of risk aversion, and so the IC continues to bind.

3.2 *Limited Liability*

Innes (1990) studies the case of limited liability and risk neutrality. The optimal contract is no longer linear and so we return to a general contract $c(V)$. He considers two versions of the model. In the first, the only restriction on the contract is limited liability on both the principal and agent, $0 \leq c(V) \leq V$. To keep the proof simple, we normalize both w and $g(0)$ to 0, although these assumptions are not necessary. Denote by $f(V, a)$ the probability density function of $V \in [0, \overline{V}]$ conditional on effort a and assume that it satisfies the monotone likelihood ratio property (MLRP), i.e.,

$$\frac{f_a(V, a)}{f(V, a)}$$

is strictly increasing in V.

The principal's problem is given by

$$\max_{c(\cdot)} \int_0^{\overline{V}} (V - c(V)) f(V, a^*)\, dV \text{ s.t.}$$

$$\int_0^{\overline{V}} c(V) f(V, a^*)\, dV - g(a^*) \geq w \tag{30}$$

$$\int_0^{\overline{V}} c(V) f_a(V, a^*)\, dV = g'(a^*) \tag{31}$$

$$0 \leq c(V) \leq V. \tag{32}$$

Note that for all contracts $c(\cdot)$ satisfying the IC (31), we have

$$\begin{aligned} &\int_0^{\overline{V}} c(V) f(V, a^*)\, dV - g(a^*) \\ &\quad \geq \int_0^{\overline{V}} c(V) f(V, 0)\, dV - g(0) \\ &\quad = \int_0^{\overline{V}} c(V) f(V, 0)\, dV \geq 0, \end{aligned}$$

where the first inequality arises because a^* maximizes the agent's utility if the IC (31) is satisfied, and the final inequality is due to the agent's limited liability, i.e., $c(V) \geq 0$. Thus, the IC (31) implies the IR (30) and so we can ignore the latter. We thus have the following Lagrangian:

$$L = \int_0^{\overline{V}} (V - c(V)) f(V, a^*)\, dV + \lambda\Big(\int_0^{\overline{V}} c(V) f_a(V, a^*)\, dV - g'(a^*)\Big),$$

which can be rewritten as

$$L = \int_0^{\overline{V}} c(V) f(V, a^*) \times \left(-1 + \lambda \frac{f_a(V, a^*)}{f(V, a^*)}\right) dV + \int_0^{\overline{V}} V f(V, a^*)\, dV - \lambda g'(a^*).$$

Pointwise optimization with respect to $c(V)$, subject to the limited liability constraint (32), yields the following contract:

$$c(V) = \begin{cases} 0 & \text{if} \quad \dfrac{f_a(V, a^*)}{f(V, a^*)} < \dfrac{1}{\lambda} \\ V & \text{if} \quad \dfrac{f_a(V, a^*)}{f(V, a^*)} \geq \dfrac{1}{\lambda} \end{cases}. \tag{33}$$

Due to MLRP, $f_a(V, a^*)/f(V, a^*)$ is strictly increasing. Thus, there exists an $\hat{X}$ such that

$$c(V) = \begin{cases} 0 & \text{if} \quad V < \hat{X} \\ V & \text{if} \quad V \geq \hat{X} \end{cases}, \tag{34}$$

where $\hat{X}$ is the largest X that satisfies the IC (31), which can be rewritten

$$\int_X^{\overline{V}} V f_a(V, a^*)\, dV = g'(a^*).$$

Contract (34) is a "live-or-die" contract: the agent receives the entire firm value V if it exceeds a threshold $\hat{X}$, and zero otherwise. The intuition is that the tails of the distribution are most informative about whether the agent has exerted effort. Thus, the optimal contract punishes the agent as much as possible for left-tail realizations of V, and rewards him as much as possible for right-tail realizations of V. With limited liability on the agent, he can receive no less than 0 for low outputs; with limited liability on the principal, she can pay no more than the entire firm value V for high outputs.

A potentially unrealistic feature of contract (34) is that it is discontinuous: when V rises from $\hat{X} - \varepsilon$ to $\hat{X}$, the principal's payoff falls from $\hat{X} - \varepsilon$ to 0. Thus, the principal may wish to exercise her control rights on the firm to "burn" output from $\hat{X}$ to $\hat{X} - \varepsilon$ to increase her payoff. Alternatively, since the agent's pay rises more than one-for-one around this threshold, he may wish to borrow on his own account to increase output from $\hat{X} - \varepsilon$ to $\hat{X}$ because the gain in his payoff will exceed the amount he must repay.

To deter both actions, the second version of the Innes (1990) model also assumes a monotonicity constraint: the principal's payoff cannot fall with firm value ($V - c(V)$ is nondecreasing in V), so the agent's pay cannot increase more than one-for-one with firm value. Following similar steps to above, the optimal contract is very similar except that at the new cutoff $\tilde{X} < \hat{X}$, the contract jumps from 0 not to V, but only to $V - \tilde{X}$, since this is the highest payoff that does not violate the monotonicity constraint. This yields the following contract:

$$c(V) = \begin{cases} 0 & \text{if} \quad V < \tilde{X} \\ V - \tilde{X} & \text{if} \quad V \geq \tilde{X} \end{cases}, \tag{35}$$

where $\tilde{X}$ is again the largest X that satisfies the IC (31), which can be rewritten

$$\int_X^{\overline{V}} (V - X) f_a(V, a^*)\, dV = g'(a^*).$$

Contract (35) is a standard call option, where the agent receives zero if V falls below a threshold X, and the residual $V - \tilde{X}$ otherwise. The intuition is similar to contract (33): for low output $\left(V < \tilde{X}\right)$, the agent receives the lowest possible payoff (0); for high output $\left(V > \tilde{X}\right)$, he gains one-for-one with any increase in V, which is the maximum possible gain without violating monotonicity.

Contract (35) implies not only that the CEO should be paid exclusively with options, but also that his wealth–performance sensitivity is 1 $\left(\text{for } V > \tilde{X}\right)$—i.e., he gains dollar-for-dollar with any increase in firm value above $\tilde{X}$.[12] Thus, the only source of variation between CEOs is the strike price $\tilde{X}$. It is easy to show that, when the marginal cost of effort $c'(a^*)$ rises, the strike price falls in order to increase the delta of the option and maintain the agent's incentives. Hence, even though the optimal contract is no longer trivial, this model does not capture much of the cross-sectional variation in real-life CEO contracts.

In reality, while CEOs are often paid with options in practice, they also receive salary, bonuses, and stock. Moreover, they often have very few shares compared to the number of shares outstanding, meaning that they gain far less than dollar-for-dollar with any increase in firm value. This wealth–performance sensitivity differs widely across firms, which the above model does not capture. We now incorporate risk aversion, which leads to the optimal sensitivity being below 1 and differing across CEOs. In addition to the strength of incentives, these models will also derive predictions for the optimal shape of contracts—whether they should be linear or convex, and thus whether they should comprise stock or options.

[12] If there are x existing shares outstanding and the CEO is given options on y shares, his share of firm value is $\frac{y}{x+y}$ if he exercises all his options. Thus, strictly speaking, he must be given infinite options to obtain a wealth–performance sensitivity of 1.

3.3 *Risk-Averse Agent*

Returning to the case of unlimited liability, another route to a meaningful contract is to have a risk-averse agent. Under the general-utility function (10), and returning to general (rather than linear) contracts, the agent's first-order condition is given by

$$(36)\quad E\left[u'(\cdot)(v'(c)c'(V)b(S) - g'(a^*))\right] = 0.$$

Even assuming a given implemented action a^*, the contracting problem remains difficult because equation (36) only requires the contract to satisfy the agent's incentive constraint *on average*. Even in the simplest case in which u is linear, the agent's *average* expected marginal benefit from effort, $E[v'(c)c'(V)b(S)]$, must equal the (known) marginal cost of effort, $g'(a^*)$. There are many potential contracts that will satisfy the incentive constraint on average, and so the problem is complex because the principal must solve for the one contract out of this continuum that minimizes the expected wage. (The problem is more complex if u is nonlinear.)

3.3.1 *Holmstrom–Milgrom Framework*

Holmstrom and Milgrom (1987, hereafter cited as HM) showed that the contracting problem becomes substantially simpler if four assumptions are made. First, the agent exhibits exponential utility, so $u(x) = -e^{-\eta x}$, where η is the coefficient of absolute risk aversion. Second, the cost of effort is pecuniary, so $v(c) = c$. Third, the noise ε is Normal, i.e., $\varepsilon \sim N\left(0, \sigma^2\right)$. Fourth, they consider a multi-period model where the agent chooses his effort every instant in continuous time. Under these assumptions, HM show that the optimal contract is linear, i.e., $c = \phi + \theta V$, and that the problem is equivalent to a single-period static problem. The intuition is that a linear contract subjects the

agent to a constant incentive pressure irrespective of the history of past performance. This result suggests that incentives should be implemented purely with stock, and not nonlinear instruments such as options.

If we also assume a quadratic cost of effort ($g(a) = \frac{1}{2}ga^2$) for simplicity, the principal's problem becomes

$$\max_{a^*,\phi,\theta} E[V - c] \tag{37}$$

$$\text{s.t.} \quad E\left[-e^{-\eta\left[c-\frac{1}{2}ga^{*2}\right]}\right] \geq -e^{-\eta w} \tag{38}$$

$$a^* \in \arg\max_{\hat{a}} E\left[-e^{-\eta\left[c-\frac{1}{2}g\hat{a}^2\right]}\right]. \tag{39}$$

Substituting for $c = \phi + \theta V$ and $V = S + b(S)a + \varepsilon$, the agent's objective function simplifies to

$$-e^{-\eta\hat{c}(a)}, \tag{40}$$

where $\hat{c}(a) = \phi + \theta(S + b(S)a) - \frac{1}{2}ga^2 - \frac{\eta}{2}\theta^2\sigma^2$ is his utility from the contract. It comprises the expected wage $\phi + \theta(S + b(S)a)$, minus the cost of effort $\frac{1}{2}ga^2$, minus the risk premium $\frac{\eta}{2}\theta^2\sigma^2$ that the agent requires. This risk premium is increasing in the agent's risk aversion η, risk σ^2, and incentives θ. From (40), the agent's first-order condition is given by

$$a^* = \frac{\theta b(S)}{g}. \tag{41}$$

His effort choice is independent of risk σ^2 and risk aversion η, since noise is additive. It is also independent of the fixed wage ϕ, since exponential utility removes wealth effects. Thus, ϕ can be adjusted to satisfy the agent's participation constraint without affecting his incentives.

Plugging (41) into the principal's objective function (37) and setting the participation constraint (38) to bind, the optimal level of incentives is (see Appendix A for a full proof):

$$\theta = \frac{1}{1 + g\eta\left(\frac{\sigma}{b(S)}\right)^2}. \tag{42}$$

Optimal incentives θ are a trade-off between two forces. A sharper contract increases effort $a = \frac{\theta b(S)}{g}$ and thus firm value, but also increases disutility $\frac{1}{2}ga^2$ and the risk premium $\frac{\eta}{2}\theta^2\sigma^2$. Thus, θ is decreasing in risk aversion η and risk σ^2 as these augment the risk premium required. The effect of the cost of effort g is more nuanced. On the one hand, fixing a^*, the required incentives are $\theta = \frac{a^* g}{b(S)}$ and increasing in g. On the other hand, when effort is costlier to implement (g is higher), the optimal effort level a^* is lower. The second effect dominates: when effort is costlier, an increase in θ leads to a smaller rise in effort, and so the optimal θ falls. (Since the benefit of effort $b(\cdot)$ has the opposite effect of the cost of effort g, we discuss only the latter throughout.)

To find fixed pay ϕ, we set the participation constraint to bind ($\hat{c}(a) = w$). This yields

$$\phi = w - \theta S - \frac{1}{2}\frac{(\theta b(S))^2}{g} + \frac{\eta}{2}\theta^2\sigma^2.$$

The comparative statics for ϕ are ambiguous (see Appendix A). On the one hand, a higher cost of effort g, higher risk aversion η, and higher risk σ^2 increase the required fixed pay ϕ as a compensating differential (i.e., to ensure the IR remains satisfied). On the other hand, these changes also reduce the optimal level of incentives (from (42)), which lowers the risk premium.

The HM framework is attractive for a number of reasons. First, it derives (rather than assumes) a linear contract as being optimal. Second, it solves for not only the optimal contract to implement a given effort level, but also the optimal effort level, i.e.,

both stages of Grossman and Hart (1983). Solving for the optimal effort level is valuable not so much because empiricists test the model's predictions for the effort level (which is hard to observe), but, as will be made clear shortly, endogenizing the effort level leads to different predictions for the contract (which is observed). Third, the fixed salary ϕ does not affect the agent's effort choice. Thus, changes in reservation utility can be simply met by varying ϕ, without changing incentives.

However, HM stressed that a number of assumptions were necessary for their linearity result: exponential utility, a pecuniary cost of effort, normal noise, and continuous time. Hellwig and Schmidt (2002) show that linearity continues to hold in discrete time under two additional assumptions: the principal does not observe the time path of profits (only the total profit in the final period); and the agent can destroy profits before he reports them to the principal. In Appendix B we discuss the role played by the first three assumptions.

The HM model has proven extremely influential due to its tractability. Given the benefits of tractability, researchers have attempted to achieve tractability in other settings. We explore these alternative models here.

3.3.2 *Fixed-Target Action*

In HM, the effort level $a = \frac{\theta b(S)}{g}$ is chosen endogenously. As described in section 3.1, an alternative specification is for the action space to be bounded above by $\overline{a}$ and the principal to implement a fixed target action $\overline{a}$. The optimal contract is now $\theta b(S) = g\overline{a}$, which leads to very different empirical implications. Now, the level of incentives $\theta b(S)$ arises from the desire to induce effort $\overline{a}$, and not any trade-off with disutility or risk. Thus, only the first effect of g exists—a higher cost of effort raises the incentives required to induce $\overline{a}$—and so incentives are increasing in g, in contrast to HM. It is also increasing in the target effort $\overline{a}$, but independent of η and σ^2, since the contract is not determined by any trade-off with these parameters. Thus, if the fixed-action model accurately represents reality, it has the attractive practical implication that the contract does not depend on the agent's risk aversion, which is typically hard to observe. It thus offers a potential explanation for why real-world contracts do not seem as complicated and contingent on as many details of the environment as standard contract theories would suggest. For example, section 3.5 shows that there is no systematic relationship between incentives and risk; the textbook by Bolton and Dewatripont (2005, p. 158) notes that "what is surprising is the relative simplicity of observed managerial compensation packages given the complexity of the incentive problem." These details do not matter because the contract is determined by the need to induce effort $\overline{a}$, rather than a trade-off with risk. In addition, we now have unambiguous predictions for how increases in risk σ^2 and risk aversion η affect the level of pay. There is now only the direct effect, that pay rises as a compensating differential, but no indirect effect because these parameters do not affect the optimal effort level.

Whether the endogenous or fixed-action model is more realistic depends on the setting. In many cases, the endogenous action case is more accurate as principals choose to implement less-than-full effort to save on wages. For example, a factory boss may only require a production operative to work an eight-hour day, to avoid paying overtime. However, for CEOs, a fixed action may be more appropriate. Edmans and Gabaix (2011b) show that, if CEO effort has a multiplicative effect on firm value, implementing full productive efficiency $\overline{a}$ is optimal if the firm is large enough. (The result also holds for any increasing function $b(S)$.) The benefits of effort are a function of firm size; the cost of effort (a higher wage

to compensate for risk and disutility) is a function of the CEO's reservation wage w. Thus, if S is sufficiently large compared to w, the benefits of effort dominate the trade-off and it is optimal to induce full productive efficiency regardless of g, η, or σ^2. For example, in a \$10 billion firm, if implementing effort level $\bar{a} - \xi$ rather than $\bar{a}$ reduces firm value by only 0.1 percent, this translates into \$10 million. If the CEO's salary is \$10 million, then even if salary can be reduced by 50 percent by allowing the CEO to exert only $\bar{a} - \xi$, implementing $\bar{a}$ remains optimal. Indeed, the structural estimation of Margiotta and Miller (2000) finds that the costs of inducing effort are substantially less than the benefits. The fixed-action model more likely applies to CEOs than rank-and-file employees, who have a limited effect on firm value.

The overall point that we would like to stress is not that one model is superior to the other. Different models apply to different scenarios. Rather, we wish to highlight how a contracting model's empirical implications hinge critically on the assumptions—whether we specify multiplicative versus additive production or preference functions, or a fixed versus continuous implemented action. Sometimes, researchers may assume a binary action space or additive functions out of convenience, but this modeling choice can lead to vastly different predictions.

3.3.3 *Noise before Action*

The framework of Edmans and Gabaix (2011b, cited as EG hereafter) provides another way to obtain tractable contracts, without the need to assume exponential utility, a pecuniary cost of effort, or normal noise. It considers the implementation of a given effort level a^*, i.e., the first stage of Grossman and Hart (1983), and thus is particularly applicable to CEOs where $a^* = \bar{a}$ if the firm is large. EG specify the noise ε as being realized before, rather than after, the action a is taken. This timing is also featured in models in which the agent observes total cash flow before deciding how much to divert (e.g., Lacker and Weinberg 1989; Biais et al. 2007; DeMarzo and Fishman 2007) and in which he observes the "state of nature" before choosing effort (Harris and Raviv 1979; Sappington 1983; Baker 1992; and Prendergast 2002). Note that this timing assumption does not render the CEO immune to risk, because noise is unknown when he signs his contract. EG also show that the contract retains the same form in continuous time, where noise and effort occur simultaneously. This consistency suggests that, if underlying reality is continuous time, it is best approximated in discrete time by modeling noise before effort.

The timing assumption allows for significant tractability. Since the noise is known when the agent takes his action, we can remove the expectation from his objective function (10) to yield

$$u\big(v(c(S + b(S)a + \varepsilon)) - g(a)\big). \tag{43}$$

In turn, $u(\cdot)$ also drops out. The specific form of u is irrelevant—since it is monotonic, it is maximized by maximizing its argument. This yields the first-order condition:

$$v'\big(c\big(S + b(S)a^* + \varepsilon\big)\big) \times c'(S + b(S)a^* + \varepsilon)b(S) = g'(a^*). \tag{44}$$

This first-order condition must hold for every possible ε, i.e., *state-by-state*, rather than simply on average. This pins down the slope of the contract: for all ε, the agent must receive a marginal felicity of $g'(a^*)$ for a marginal increase in V. Thus, for all V, the contract must satisfy

$$v'(c(V))c'(V)b(S) = g'(a^*).$$

Integrating over 0 to V, we obtain in felicity units,

$$v(c(V)) = \frac{g'(a^*)}{b(S)} V + k,$$

for an integration constant k. This yields, in dollar terms,

$$c(V) = v^{-1}\left(\frac{g'(a^*)}{b(S)} V + k\right). \tag{45}$$

The constant k is chosen to make the participation constraint bind, i.e.,

$$E\left[u\left(\frac{g'(a^*)}{b(S)} V + k - g(a^*)\right)\right] = w. \tag{46}$$

There is a unique optimal contract. The slope is chosen so that the incentive constraint (44) holds state by state, and the scalar k is chosen so that the participation constraint binds.

Equation (45) shows that the optimal contract is typically nonlinear. Even though the noise is known when the agent takes his action, it is not irrelevant because it has the potential to undo the agent's incentives. If ε is high, V and thus c will already be high; a high reservation wage w increases the required constant k and thus c, and so has the same effect. If the agent exhibits diminishing marginal felicity (i.e., v is concave), he has lower incentives to exert effort. Put differently, the agent does not face *risk* (as ε is known) but *distortion* (as ε affects his effort incentives). HM assume that the cost of effort is in financial terms so that, like the benefit of effort, it also declines with ε, and so incentives are unchanged with a linear contract. EG instead address distortion by the shape of the contract: it is convex, via the v^{-1} transformation. If noise is high, the contract gives a greater number of dollars for each incremental unit of firm value ($c'(V)$), to offset the lower marginal felicity of each dollar ($v'(c)$). Therefore, the marginal felicity from effort remains $v'(c)c'(V)b(S) = g'(a^*)$, and incentives are preserved regardless of w or ε. Allowing for convex contracts removes the need to assume a pecuniary cost of effort. In contrast, if v is convex, the contract is concave.

The contract is linear in two special cases. The first is a pecuniary cost of effort, as in HM: when $v(c) = c$, we have $c(V) = \frac{g'(a^*)}{b(S)} V + k$. With an additive production function ($\gamma = 0$), the CEO's dollar incentives are linear in the firm's dollar value V; with a multiplicative production function ($\gamma = 1$), they are linear in the firm's percentage return $\frac{V}{S}$. The former result echoes Lacker and Weinberg (1989), who also feature a pecuniary cost of effort and an additive production function. They show that the optimal contract to deter all cash flow diversion (the analogy of $a^* = \bar{a}$) is piecewise linear. The second case is $v(c) = \ln c$, i.e., multiplicative preferences. The contract is $\ln c(V) = \frac{g'(a^*)}{b(S)} V + k$, and so log pay is linear in $V\left(\frac{V}{S}\right)$ with an additive (multiplicative) production function. In both cases, the framework delivers linear contracts without requiring exponential utility or Normal noise. More broadly, the framework allows for contracts that are convex and concave, rather than purely linear as in HM—thus, tractability can be achieved without linearity—and shows what determines the optimal curvature or linearity of the contract: the form of $v(\cdot)$.

Equation (45) also clarifies the parameters that do and do not matter for the contract's functional form. It depends only on the felicity function v and the cost of effort g. The functional form is independent of the utility function u, the reservation utility w, and the distribution of the noise ε, i.e., the contract can be written without reference to these parameters. These parameters will still affect the contract's slope via their impact on

the scalar k. However, the contract's slope as well as its functional form are independent of u, w, and ε in the cases of $v(c) = c$ and $v(c) = \ln c$, where it depends only on $g'(a^*)$. This "detail independence" contrasts with standard agency models where the contract depends on many specific features of the setting.[13]

The above framework allows for tractable contracts with fewer restrictions on the utility function, cost of effort, and noise distribution, as well as nonlinear contracts. This tractability allows it to be used in dynamic models with private saving (Edmans et al. 2012) and assignment models with moral hazard under risk aversion (Edmans and Gabaix 2011a). However, it has a number of disadvantages. It requires the assumption that noise precedes the action; while applicable in some settings (e.g., a cash flow diversion model), it may not apply to others. It also assumes a fixed implemented action a^*, which again may only apply in some settings (e.g., a CEO of a large firm). The goal of this article is to provide a range of modeling frameworks, each of which may be applicable under different conditions.

3.3.4 *Constant Relative Risk Aversion and Lognormal Firm Value*

We have so far considered two models that yield tractable contracts at the cost of some assumptions. Other papers do not aim to achieve a tractable analytical solution, but instead to calibrate the optimal contract, and so use fewer assumptions. Perhaps the most commonly used framework for calibration involves constant relative risk aversion (CRRA) utility and lognormal firm value, studied by Lambert, Larcker, and Verrecchia (1991); Hall and Murphy (2000, 2002); Hall and Knox (2004); and others. We present here the version calibrated in Dittmann and Maug (2007). End-of-period firm value is given by

$$V_T = V_0(a) \exp\left[\left(R - \frac{\sigma^2}{2}\right)T + \varepsilon\sigma\sqrt{T}\right],$$

where $\varepsilon \sim N(0, 1)$ and V_0 satisfies $V_0' > 0$ and $V_0'' < 0$. They assume that a contract is composed of salary ϕ, θ shares, and ψ options (as a fraction of shares outstanding), with strike price X and maturity T.[14] Both shares and options are paid out at the end of the period; salary ϕ is paid out at the start. The CEO begins with non-firm wealth W_0, which is invested at the risk-free rate R. His end-of-period wealth is then given by

$$c_T = (\phi + W_0)e^{RT} + \theta V_T + \psi \max\{V_T - X, 0\}.$$

In the general-utility function (10), we have $u(x) = x$ and $v(c_T) = \frac{c_T^{1-\zeta}}{1-\zeta}$, where ζ is the parameter of relative risk aversion. Thus, the CEO's preferences are given by

$$U(c_T, a) = \frac{c_T^{1-\zeta}}{1-\zeta} - g(a).$$

Assuming risk-neutral pricing, the CEO's end-of-period pay (change in wealth) is given by

$$\pi_T = \phi e^{RT} + \theta V_T + \psi \max\{V_T - X, 0\},$$

with expected present value

$$\pi_0 = E\left[e^{-RT}\pi_T\right] = \phi + \theta V_0 + \psi BS,$$

where BS denotes the Black–Scholes value of the option. They solve for the first stage

[13] Chassang (2013) derives contracts that are relatively independent of the environment (i.e., the probability space), in a risk-neutral setting.

[14] In an additional analysis, Dittmann and Maug (2007) also solve for the optimal unrestricted contract.

of Grossman and Hart (1983), in which the principal wishes to implement action a^*, and so her problem is given by

$$\min_{\phi,\theta,\psi} \pi_0 = \phi + \theta V_0 + \psi BS$$

$$\text{s.t.} \quad E[U(W_T, a^*)] \geq \overline{U},$$

$$a^* = \arg\max_{a \in [0,\infty)} E[U(W_T, a)],$$

$$\phi + W_0 \geq 0, \quad 0 \leq \theta \leq 1, \quad \psi \geq 0.$$

Dittmann and Maug (2007) calibrate this model to a sample of 598 US CEOs. In particular, they study whether it is more efficient to incentivize the CEO with stock or options. Since options are riskier, \$1 of options is worth less to the CEO than \$1 of stock, rendering them less effective in meeting the CEO's participation constraint. On the other hand, \$1 of options provides greater incentives than \$1 of stock, rendering them more effective in meeting his incentive constraint. They find that the first effect is dominant, suggesting that the optimal contract should involve only stock and not options. This prediction is shared with Holmstrom and Milgrom (1987), who predict linear contracts, although in a different setting. Moreover, when they drop the restriction that the contract must be piecewise linear (i.e., consist of salary, stock, and options), they find that the optimal nonlinear contract is concave.

In contrast to both frameworks, option compensation is widespread in the United States. One interpretation, consistent with Bebchuk and Fried (2004), is that the use of options indicates rent extraction: since at-the-money options did not have to be expensed until 2006, they constitute "stealth compensation" not noticed by shareholders. Indeed, Hayes, Lemmon, and Qiu (2012) found that the use of options fell substantially after FAS 123R mandated that the economic value of an option be expensed, thus leading to an accounting charge for at-the-money options. However, Dittmann, Maug, and Spalt (2010) show that options can be rationalized if the CEO is loss averse: since options provide downside protection, they are particularly valuable to a loss-averse agent. Moreover, as we will discuss in section 3.7, if the agent chooses firm risk in addition to effort, options may be useful to induce him to take value-adding risky projects.

3.4 *Incentives in Market Equilibrium*

Section 3 has thus far taken the reservation wage w as given. We now endogenize w using the assignment model of Gabaix and Landier (2008) to study how CEO incentives vary across firms in market equilibrium. We use the Edmans, Gabaix, and Landier (2009) framework of a risk-neutral CEO, multiplicative preferences, and a fixed target action, as in section 3.1, with $a^* = \overline{a}$. We will show that even this simple model leads to predictions consistent with empirical findings. (Edmans and Gabaix 2011a extend the model to risk aversion.)

From (20), we have $\theta = \frac{\Lambda w}{S}$ where $\Lambda = g'(a^*)$. The fixed salary ϕ is chosen so that the IR binds, i.e., $\phi = w - \theta S = w(1 - \Lambda)$. Thus, the CEO in firm n is given a fixed salary ϕ_n, and $\theta_n S_n$ worth of shares, with

$$\theta_n S_n = w(n)\Lambda, \tag{47}$$

$$\phi_n = w(n)(1 - \Lambda), \tag{48}$$

where $w(n)$ is given by equation (8) from Gabaix and Landier (2008). Thus, a fraction Λ of the equilibrium wage is paid in equity, and the remainder is paid in cash.

We can now solve for the three incentive measures in equations (21)–(23) in terms of model primitives:

$$\theta^I = \Lambda \propto S^0 \tag{49}$$

$$\theta^{II} = \Lambda \frac{w}{S} \propto S^{\rho-1} \tag{50}$$

$$\theta^{III} = \Lambda w \propto S^{\rho}. \tag{51}$$

Equation (20) earlier suggested that, in a multiplicative model, the optimal incentive measure is θ^I (%–% incentives), since it affects the implemented effort level. Equations (49)–(51) illustrate a related advantage: in a multiplicative model, θ^I is independent of firm size and thus comparable across firms of different size. Intuitively, since effort has a percentage effect on both firm value and CEO utility, it is %–% incentives that are relevant. Comparability across firms of different size is useful to study which firms are incentivized more or less than their peers. For example, a passive investor who believes that incentives are not fully priced in the market may wish to invest in a stock with high CEO incentives; an activist investor may wish to target a firm with low incentives. However, if the CEO of a large firm has $2 million of equity and the CEO of a smaller firm has $1 million of equity, we cannot immediately conclude which CEO is better incentivized, as dollar equity holdings should optimally increase with firm size. Relatedly, comparability is valuable for boards or compensation consultants undertaking benchmarking analyses.[15]

While %–% incentives should be independent of size, with $\rho = \gamma - \beta/\alpha = 1/3$ as in Gabaix and Landier (2008), $–$ incentives should have a firm-size elasticity of $\rho - 1 = -2/3$. If effort has a multiplicative effect on firm value, it has a higher dollar effect in a larger firm, and so a lower equity stake is needed to induce effort. In addition, $–% incentives should have an elasticity of $\rho = 1/3$. Larger firms hire more talented CEOs who command higher wages. Since the benefits of shirking are higher, given multiplicative preferences, a higher dollar equity stake is needed to induce effort. In section 3.5, we will compare these predictions to the data.

Turning to the strength of incentives, the $–$ incentives measured by Jensen and Murphy (1990) are given by $\theta^{II} = \theta^I \frac{w}{S}$. Since firm size S is substantially larger than the CEO's wage w, $–$ incentives should be low. Because firms are so large, the dollar benefits of effort are much greater than the disutility cost to the CEO, and so only a small equity stake is needed to induce effort.

Another strand of research justifies low $–$ incentives by pointing out the disadvantages of strong incentives. Lambert, Larcker, and Verrecchia (1991) show that a high equity stake may induce the CEO to take inefficiently low risk. Benmelech, Kandel, and Veronesi (2010) assume that equity incentives vest in the short term, since long-term incentives expose the CEO to risks outside his control. Then, the CEO may conceal information that his investment opportunities have declined to keep the current stock price high, even though disclosing such information will allow him to efficiently disinvest. In a similar vein, Peng and Roell (2008, 2014) and Goldman and Slezak (2006) demonstrate that high-powered incentives that vest in the short term can encourage the manager to expend firm resources to manipulate the stock price upwards. However, these disadvantages can potentially be avoided by granting equity with long-vesting horizons.

3.5 *Empirical Analyses*

We now turn to tests of the empirical predictions of these models. The first set of tests study the level of incentives. Motivated by traditional additive models, Jensen and Murphy (1990) estimated $–$ incentives and showed that the CEO loses only $3.25

[15] By analogy, fund managers are compared according to their risk-adjusted percentage returns, rather than dollar returns, as the former is comparable across funds of different size (assuming constant returns to scale).

for every \$1,000 loss in firm value, an effective equity stake of only 0.325 percent. They interpreted this stake as too low to be reconciled with optimal contracting, and thus concluded that CEOs are "paid like bureaucrats." However, such a conclusion hinges critically on whether we believe CEO effort has additive or multiplicative effects on firm value and CEO utility. \$–\$ incentives are the relevant measure only in an additive model. As discussed above, in a multiplicative model, %–% incentives are relevant and \$–\$ incentives should optimally be low. In Hall and Liebman (1998), \$–% incentives are relevant—i.e., dollar ownership, rather than percentage ownership. They overturned Jensen and Murphy's conclusion by showing that dollar ownership is sizable.

Separately, theory predicts that incentives should be $\frac{g\bar{a}}{b(S)}$ or $\frac{1}{1+g\eta\left(\frac{\sigma}{b(S)}\right)^2}$, but parameters such as the cost of effort g are difficult to quantify. Thus, it is difficult to evaluate whether quantitative findings on the level of incentives are consistent with efficiency. Haubrich's (1994) calibration suggests that the seemingly low incentives found by Jensen and Murphy (1990) can be optimal if the CEO is sufficiently risk averse, but attaches wide confidence intervals to his conclusion, given the difficulties in calibration. The structural estimation of Margiotta and Miller (2000) also finds that low incentives are sufficient to induce effort, given the multiplicative effect of effort on firm value.

Given the difficulties of quantifying parameters such as g to calculate the optimal level of incentives, incentive theories are typically tested instead in terms of their cross-sectional predictions—whether they vary with parameters such as S, g, η, and σ^2 as predicted. Note that it is important for empirical tests to study the precise measure of incentives predicted by the theory. For example, if the theory is a multiplicative model that predicts how the dollar equity stake θS varies with g, η, and σ^2, studying the percentage equity stake θ will not be a precise test of the model as these parameters may vary with firm size S. In addition, equation (42) implies that with a multiplicative production function ($b(S) = S$), the relevant measure of risk is $\frac{\sigma}{S}$, the volatility of the firm's percentage returns; with an additive production function ($\gamma = 0$) it is σ, the volatility of the firm's dollar returns.

Starting with size, Jensen and Murphy (1990) find that \$–\$ incentives are even lower in large firms, perhaps because governance is particularly weak in these firms. As discussed above, Edmans, Gabaix, and Landier (2009) show that, under a multiplicative model, CEO effort has a larger dollar effect in a bigger firm, and so a smaller equity stake is required to induce effort. They quantitatively predict a firm-size elasticity of $-2/3$, consistent with their empirical finding of -0.60. Similarly, they find that \$–\$ incentives are independent of firm size and \$–% incentives have a size elasticity of 1/3, both as predicted. Thus, a model with multiplicative utility and production functions quantitatively explains the size scalings of incentives. While these results are consistent with incentives being set optimally and the true model indeed being multiplicative, they could also be consistent with a non-multiplicative model and suboptimal incentive setting.

We now turn to HM's prediction that incentives θ are decreasing in risk σ. While Lambert and Larcker (1987), Aggarwal and Samwick (1999), and Jin (2002) indeed find a negative relationship, Demsetz and Lehn (1985), Core and Guay (1999), Oyer and Schaefer (2005), and Coles, Daniel, and Naveen (2006) document a positive relationship, and Garen (1994), Yermack (1995), Bushman, Indjejikian, and Smith (1996), Ittner, Larcker, and Rajan (1997), Conyon and Murphy (2000), Edmans, Gabaix, and Landier

(2009), and Cheng, Hong, and Scheinkman (2015) show either no relationship or mixed results. Aggarwal and Samwick (1999) and Jin (2002) study the volatility of dollar returns, and the other papers study percentage returns. Thus, the empirical evidence points to a weak relationship between risk and incentives. The fixed-action model provides a potential explanation: risk is second order compared to the benefits of effort—it is incentive considerations, not risk considerations, that affect the slope of the contract.[16]

The prediction that θ is decreasing in risk aversion η is harder to test as risk aversion is unobservable. Becker (2006) uses data on CEO wealth, available in Sweden, as a (negative) proxy for risk aversion under the assumption of decreasing absolute risk aversion. As predicted, he finds that wealth is positively related to both \$–\$ and %–% incentives.[17] In addition, wealth can affect incentives through channels other than risk aversion. In the EG model, where the contract is not driven by a trade-off with risk aversion, the CEO's outside option w may include consuming his existing wealth. Higher wealth increases w and thus the constant k (equation (46)), which in turn augments incentives (equation (45)). Intuitively, if the CEO is wealthier, his marginal utility from money is lower, and so greater incentives are required to induce him to work. More generally, while studies have shown that incentives are significantly related to determinants such as firm size, risk, and wealth, Coles and Li (2013) find that a large portion is explained by manager fixed effects, similar to Graham, Li, and Qiu (2012) who find significant manager fixed effects for pay levels. Thus, a sizable component of the variation in incentives remains unexplained.

The theories also derive predictions for expected pay $E[c]$, often referred to as the level of pay. As discussed, firm risk and disutility have an ambiguous effect on the level of pay in the HM model, but increase it in the fixed-action model due to the required compensating differential. Garen (1994) shows that pay is insignificantly increasing in firm risk as measured by dollar volatility, and insignificantly decreasing in percentage volatility. Cheng, Hong, and Scheinkman (2015) find a significant positive relationship with percentage volatility for financial firms. Gayle and Miller (2009) show theoretically and empirically that larger firms are more complex to manage, and so CEOs require greater pay in return. In addition, greater agency problems in large firms necessitate higher equity incentives, and thus more pay as a risk premium. Conyon, Core, and Guay (2011) and Fernandes et al. (2013) compare CEO pay in the United States to the rest of the world, and show that the pay premium to US CEOs can be explained by the greater risk that they bear, rather than rent extraction. The structural estimation of Gayle, Golan, and Miller (2015) finds that the risk premium can explain over 80 percent of the pay differential between small and large firms. It arises both because large firms require greater incentives to address moral hazard, and also because stock returns are a poorer signal of effort in large firms.

3.6 *Multiple Signals*

The analysis has thus far studied the sensitivity of the manager's pay to the performance

[16] Prendergast (2002) provides another explanation for the weak relationship between risk and incentives. When uncertainty is low, principals assign tasks to agents and directly monitor them. When uncertainty is high, they delegate tasks to agents and incentivize them through output. His model applies principally to rank-and-file employees, since day-to-day monitoring of the CEO by directors is more limited.

[17] While HM assume constant absolute risk aversion utility and so risk aversion is independent of wealth, the model of Sannikov (2008), analyzed in section 4.2, generally predicts that incentives fall with risk aversion by the same intuition as in HM. His model allows for general-utility functions, and thus absolute risk aversion to be decreasing in wealth.

of his own firm. Here, we study the extent to which it should depend on other signals, such as industry and market conditions. We first demonstrate the Holmstrom (1979) informativeness principle. This principle considers the case in which, in addition to firm value V, the principal has access to an additional contractible signal z (such as the performance of peers) and studies the extent to which CEO pay c should depend on z. The joint density function is given by $f(V, z, a)$, and the principal's problem is

(52)

$$\max_{c(\cdot,\cdot)} \int_z \int_0^{\overline{V}} (V - c(V, z)) f(V, z, a^*)\, dV\, dz$$

s.t.

(53)

$$\int_z \int_0^{\overline{V}} u(c(V, z)) f(V, z, a^*)\, dV\, dz \geq g(a^*),$$

(54)

$$\int_z \int_0^{\overline{V}} u(c(V, z)) f_a(V, z, a^*)\, dV\, dz = g'(a^*).$$

Denote by λ and μ the Lagrange multipliers for the IR (53) and IC (54), respectively. Pointwise optimization yields the following condition for the optimal contract $c(V, z)$:

$$\frac{1}{u'(c(V, z))} = \lambda + \mu \frac{f_a(V, z, a^*)}{f(V, z, a^*)}$$

for all (V, z). Hence, assuming that $\mu \neq 0$ (i.e., the IC binds), the contract $c(V, z)$ is not a function of z if and only if the likelihood ratio $f_a(V, z, a^*)/f(V, z, a^*)$ does not depend on z, i.e.,

(55) $$\frac{f_a(V, z, a^*)}{f(V, z, a^*)} = h(V, a^*)$$

for some function h. Condition (55) holds if and only if V is a sufficient statistic for $\{V, z\}$ with respect to $a = a^*$.[18] Thus, any signal z, no matter how noisy, that provides information incremental to V on the agent's effort choice, should be included in the contract.

The most common application of the informativeness principle to CEO pay is relative performance evaluation (RPE). Specifically, peer performance is informative about the degree to which high firm value V is due to high effort or good luck and so should enter the contract. For example, CEO pay should be based on performance relative to a peer group, rather than using standard stock and options whose value is based on absolute performance.

However, Holmstrom's result was derived assuming optimal contracts. In reality, contracts may not be perfectly optimal. For example, a preference for simplicity can lead to the use of piecewise linear contracts—indeed, cash, stock, and options are typically used in practice.[19] Dittmann, Maug, and Spalt (2013) study the effect of indexation when contracts are restricted to these instruments and show that the indexation of options can destroy incentives. Since an indexed option is in the money only if the stock price rises high enough to outperform the benchmark, indexation is tantamount to increasing the strike price of an option and reducing the drift rate of the underlying asset. Both effects reduce the option's delta and thus his incentives. To preserve incentives, additional equity must be given, and their calibration

[18] Condition (55) is required to be satisfied only for effort level $a = a^*$; it may be that the likelihood ratio depends on z for a different $a \neq a^*$. We thus say that (55) holds if and only if V is a sufficient statistic for $\{V, z\}$ with respect to $a = a^*$. Holmstrom (1979) assumes that condition (55) is satisfied either for all a or no a, and so he shows that the contract is a function of z if and only if V is a sufficient statistic for $\{V, z\}$ with respect to a (rather than $a = a^*$).

[19] See Gabaix (2014) for a sparsity-based model where agents have a preference for simplicity.

shows that full indexation of all options would increase compensation costs by 50 percent, on average. If firms choose the optimal proportion of options to index, average compensation costs would only fall by 2.3 percent, and 75 percent of firms would choose zero indexation. They show that indexing stock also has little benefit. Chaigneau, Edmans, and Gottlieb (2016a) study limited liability, another contracting constraint relevant in reality, and show that the informativeness principle may no longer hold. In the standard Innes (1990) framework, the agent receives zero below a threshold and gains one-for-one above the threshold, which is the maximum possible without violating the agent's monotonicity constraint (if imposed) or principal's limited liability (if monotonicity is not imposed). Since constraints on the contract are binding almost everywhere, the principal's ability to use the signal is severely restricted. If low firm value V is accompanied by a low signal z, she cannot punish the agent further without violating limited liability, as he is already receiving zero. Her only degree of freedom is on the level of the threshold, and so a signal is only valuable if it affects the optimal threshold. Moreover, Chaigneau, Edmans, and Gottlieb (2016b) show that, even if a signal has strictly positive value because it affects the optimal cutoff, its usage can also weaken incentives (similar to Dittmann, Maug, and Spalt 2013), and so its value is quantitatively small.

In addition, other concerns can lead to pay optimally depending on industry performance. Oyer (2004) shows that, if equity is forfeited upon departure, it induces the agent to stay with the firm. Since non-indexed equity is more valuable in high market conditions, when the outside option is also higher, its retention power increases precisely when retention concerns are greatest. Gopalan, Milbourn, and Song (2010) argue that not indexing an executive to industry performance induces him to choose the firm's industry exposure optimally. DeMarzo and Kaniel (2015) and Liu and Sun (2015) show that, when CEOs have relative wealth concerns, it is optimal for the firm to pay him for general industry upswings to ensure that his pay does not lag his industry peers.

Turning to the evidence, conventional wisdom is that RPE is very rarely used in reality. Aggarwal and Samwick (1999) and Murphy (1999) show that CEO pay is determined by absolute, rather than relative, performance, and Jenter and Kanaan (2015) find an absence of RPE in CEO firing decisions. However, more recent evidence suggests that RPE may be more common than previously thought. Albuquerque (2009) argues that relevant peers are not only firms in the same industry, but also those of similar size, since common external shocks may affect different firms in the same industry in different ways—for example, increases in regulation may be particularly costly for small firms. When defining firms according to both industry and size, rather than industry alone, she finds significant evidence for RPE. Gong, Li, and Shin (2011) argue that conclusions that RPE is rare arise from identifying RPE based on an implicit approach—assuming a peer group (e.g., one based on industry and/or size) and relevant performance measures, and studying whether those performance measures for that peer group affect CEO pay. These assumptions may lead to measurement error that biases downwards the estimated use of RPE. Gong et al. study the explicit use of RPE, based on the disclosure of peer firms and performance measures mandated by the SEC in 2006. They find that 25 percent of S&P 1500 firms explicitly use RPE. Similarly, rather than assuming a peer group, Lewellen (2015) hand-collects the peers that firms report as their primary product market competitors in their 10-K filings, and finds evidence for RPE. DeAngelis and Grinstein (2016) examine the actual terms of compensation contracts and find

that 88 percent of RPE contracts measure the rank performance of the CEO relative to peers. In contrast, most empirical studies measure the difference between firm performance and a peer–firm benchmark, implicitly assuming that contracts concern absolute peer-adjusted performance. Using a rank-based specification, they find significant evidence of RPE. However, while recent evidence suggests that RPE is not rare, it is still far less common than the universality that frictionless models would predict.

In addition to signals about peer performance, the informativeness principle implies that any informative signal, no matter how noisy, should be in the optimal contract. In reality, in addition to firm value, CEO pay may depend on accounting performance measures (such as sales growth, return-on-assets, and earnings per share growth) through their impact on discretionary bonuses, performance-based vesting provisions (Bettis et al. 2010), and subjective evaluations by principals (e.g., Cornelli, Kominek, and Ljungqvist 2013). However, CEO pay does not appear to depend on non-accounting performance measures such as surveys on intangible assets (e.g., customer satisfaction, brand strength, and employee engagement) and the number of patent citations. These all potentially provide information over and above that contained in the stock price, since the stock market does not immediately capitalize intangibles (Edmans 2011).

3.7 *Risk Taking*

Thus far, the CEO takes an action that changes the firm's expected value, but has no direct effect on its risk. In Smith and Stulz (1985), the agent takes a single action that reduces risk via hedging. If the agent is risk averse, he will engage in excessive hedging; in a CEO context, this corresponds to turning down positive NPV risky projects. They show how options address this issue, since their convexity counterbalances the concavity of the agent's utility function. Dittmann, Yu, and Zhang (2015) calibrate a model where the CEO chooses both effort and risk, and show that it can explain the mix of stock and options found empirically. However, Carpenter (2000) and Ross (2004) show theoretically that options may not increase the manager's risk-taking incentives: while an option has "vega" (positive sensitivity to volatility), it also has "delta" (positive sensitivity to firm value). Thus, a risk-averse manager may wish to reduce volatility in the value of the firm and thus his options. Shue and Townsend (forthcoming b) evaluate this theoretical debate empirically by showing that exogenous increases in options, resulting from their multiyear grant cycles, lead to an increase in risk taking.

The above models consider "good" risk taking that improves firm value. However, the CEO may also have incentives to engage in "bad" risk taking that reduces firm value. In particular, in a levered firm, an equity-aligned manager may undertake a project even if it is negative NPV because shareholders benefit from the upside, but have limited downside risk due to limited liability (Jensen and Meckling 1976). Anticipating this, creditors will demand a high cost of debt and/or tight covenants, to the detriment of shareholders.

Edmans and Liu (2011) show that a potential solution to such risk shifting is to compensate the CEO with debt as well as equity. (Such debt is referred to as "inside" debt, as it is owned by the manager, rather than outside creditors.) Previously proposed remedies for risk shifting include bonuses for achieving solvency, or salaries and private benefits that are forfeited in bankruptcy (e.g., Brander and Poitevin 1992). These instruments are sensitive to the incidence of bankruptcy, but if bankruptcy occurs, they pay zero regardless of liquidation value. In contrast, inside debt yields a positive payoff in bankruptcy proportional to the recovery

value. Thus, it renders the manager sensitive to firm value in bankruptcy, and not just the incidence of bankruptcy—exactly as desired by creditors—and thus reduces the cost of raising debt, to the benefit of shareholders. Indeed, recent empirical studies have shown that CEOs hold a substantial amount of inside debt through defined-benefit pensions and deferred compensation. These are unsecured obligations that yield an equal claim with other creditors in bankruptcy, and thus constitute inside debt. For example, Sundaram and Yermack (2007) show that GE's Jack Welch had over $100 million of inside debt when he retired in 2001.

Since traditional contracting theories typically advocate only the use of equity, and disclosure of pensions and especially deferred compensation was limited prior to a 2006 SEC disclosure reform, Bebchuk and Fried (2004) argue that inside debt constitutes "stealth compensation through retirement benefits." However, the risk deterrence story suggests that inside debt can be consistent with optimal contracting. The disclosure of significant inside debt positions following the SEC reform led to an increase in bond prices (Wei and Yermack 2011), and is associated with lower bond yields and fewer covenants (Anantharaman, Fang, and Gong 2014, using personal state income taxes as an instrument for inside debt). Debt-aligned executives manage the firm more conservatively, as measured by the firm's lower distance to default (Sundaram and Yermack 2007), and lower stock-return volatility, R&D expenditures, and financial leverage (Cassell et al. 2012, also using the income tax instrument). Indeed, the alignment of executives with debt has gathered pace in the recent crisis. In 2010, American International Group tied 80 percent of highly paid employees' pay to the price of its bonds, and 20 percent to the price of its stock, and UBS and Credit Suisse have since started paying bonuses in bonds. The Liikanen Report of the European Commission and the Federal Reserve have advocated debt-like compensation to curb excessive risk taking. However, even if the above studies can be interpreted as showing causal effects of inside debt on risk taking and borrowing conditions, they do not study whether shareholders benefit overall, nor whether alternative solutions to risk shifting would be superior.

In Smith and Stulz (1985), the firm is unlevered so there are no risk-shifting concerns; the contract contains options but no debt. In Edmans and Liu (2011), the CEO is risk neutral, so there is no problem of inducing him to take "good" risk; the contract contains debt, but not options. For future research, it would be interesting to incorporate both leverage and risk aversion into a model of both effort and risk taking, to study the optimal mix of salary, stock, options, and debt.

4. *Dynamic Incentives*

This section analyzes dynamic models of moral hazard. In reality, CEOs are employed for several years. A dynamic setting leads to additional questions, such as how to spread the rewards for good performance over time, how the level and sensitivity of pay vary over time, and when the CEO quits or is fired. We start in section 4.1 with a tractable discrete-time model that yields closed-form solutions, at the cost of some assumptions. In section 4.2 we move to a continuous-time model which typically yields numerical solutions, but allows for departures and terminations.

4.1 *Dynamic Incentives: A Simple Discrete Time Model*

We present here the discrete-time version of the Edmans et al. (2012) model, which uses the EG framework to yield tractable solutions. In every period t, the CEO first observes noise ε_t and then takes action $a_t \in [0,\overline{a}]$,

which affects terminal (period-T) firm value as follows:

$$V_T = Se^{\sum_{s=1}^{T}(a_s+\varepsilon_s)}.$$

We assume that a signal about a_t, $V_t = Se^{\sum_{s=1}^{t}(a_s+\varepsilon_s)}$, is contractible. The incremental information contained in V_t over and above that contained in V_{t-1} can be summarized by the stock "return"

$$r_t = \ln V_t - \ln V_{t-1} = a_t + \varepsilon_t,$$

where, as in EG, the CEO observes the noise ε_t before he takes his action a_t. Also in every period t, the principal pays the CEO $y_t(r_1, \ldots, r_t)$ which may depend on the entire history of returns. The agent consumes c_t and saves $(y_t - c_t)$ (which may be positive or negative) at the continuously compounded risk-free rate R. We consider two versions of the model. In one, private saving is observed by the principal and so she can stipulate that $y_t = c_t$, i.e., that the CEO does not engage in private saving. In another, private saving is unobserved. This leads to additional complications, since the CEO may have incentives to engage in a joint deviation of simultaneously shirking and saving—by saving, he insures against future income shocks, thus reducing effort incentives. Put differently, by privately saving, the CEO can achieve a different consumption profile c_t from the income y_t provided by the contract, thus undoing effort incentives. The contract must therefore remove his incentives to do so.

The CEO lives for T periods and retires after period $L \leq T$.[20] His lifetime utility is

$$U = \sum_{t=1}^{T} e^{-\delta t}\mathsf{u}(c_t, a_t), \tag{56}$$

$$\mathsf{u}(c, a) = \ln c - g(a),$$

where δ is the agent's discount rate, i.e., his impatience. His per-period utility function $\mathsf{u}(c, a) = \ln c - g(a)$ corresponds to (10) with $v(c) = \ln c$ and $u(x) = x$, i.e., multiplicative preferences. The agent's reservation utility is w.

The principal is risk-neutral and so her objective function is expected discounted terminal firm value minus expected pay:

$$\max_{(a_t, t=1,\ldots,L),(y_t, t=1,\ldots,T)} E\left[e^{-RT}V_T - \sum_{t=1}^{T} e^{-Rt}y_t\right]. \tag{57}$$

She wishes to implement a target action sequence (a_t^*).

There are two constraints to consider. The first is the effort constraint (EF), which ensures that the CEO does not wish to deviate from (a_t^*). We consider a local deviation in the action a_t after history $(r_1, \ldots, r_{t-1}, \varepsilon_t)$. The effect on CEO utility should be zero:

$$0 = E_t\left[\frac{\partial U}{\partial r_t}\frac{\partial r_t}{\partial a_t} + \frac{\partial U}{\partial a_t}\right].$$

Since $\partial r_t/\partial a_t = 1$ and $\partial U/\partial a_t = e^{-\delta t} \times \mathsf{u}_a(c_t, a_t)$, the EF constraint (evaluated at $a_t = a_t^*$) is

$$\text{EF}: E_t\left[\frac{\partial U}{\partial r_t}\right] = e^{-\delta t}\mathsf{u}_a(c_t, a_t^*) \quad \text{if} \quad a_t^* \in (0, \overline{a}); \tag{58}$$

$$E_t\left[\frac{\partial U}{\partial r_t}\right] \geq e^{-\delta t}\mathsf{u}_a(c_t, a_t^*) \quad \text{if} \quad a_t^* = \overline{a},$$

for $t \leq L$.

The second constraint is the private savings constraint (PS), which ensures that the CEO consumes his income in period t, i.e., $c_t = y_t$, so that he has no incentive to save privately. If the CEO saves a small amount d_t in period t and invests it until $t + 1$, his

[20] In the model, the principal replaces the CEO with a new one and continues to contract optimally, but this assumption can easily be weakened.

utility increases to the leading order by $-E_t\left[\frac{\partial U}{\partial c_t}\right] d_t + E_t\left[\frac{\partial U}{\partial c_{t+1}}\right] e^R d_t$. To deter private saving or borrowing, this change should be zero to the leading order,

$$\text{(59)} \quad \text{PS}: 1 \;=\; E_t\left[e^{R-\delta}\,\frac{\mathsf{u}_c(c_{t+1}, a_{t+1})}{\mathsf{u}_c(c_t, a_t)}\right],$$

that is, the consumption Euler equation should hold. If the CEO cannot engage in private saving (e.g., because the principal can observe saving), then instead the inverse Euler equation (IEE) holds, as is standard in agency problems with additively separable utility (e.g., Rogerson 1985 and Farhi and Werning 2012). This is given by

$$\text{(60)} \quad \text{IEE}: 1 \;=\; E_t\left[\frac{1}{e^{R-\delta}}\,\frac{\mathsf{u}_c(c_t, a_t)}{\mathsf{u}_c(c_{t+1}, a_{t+1})}\right].$$

We next present the solution (a heuristic proof is in Appendix A). For simplicity, we assume a constant target action ($a_t^* = a^*$, which may correspond to full productive efficiency $\overline{a}$). The contract is given by

$$\text{(61)} \qquad \ln c_t \;=\; \ln c_0 + \sum_{s=1}^{t} (\theta_s r_s + k_s),$$

where θ_s and k_s are constants. The sensitivity θ_s is given by

(62)

$$\theta_s \;=\; \begin{cases} \dfrac{g'(a^*)}{1 + e^{\delta} + \cdots + e^{\delta(T-s)}} & \text{for} \quad s \le L \\ 0 & \text{for} \quad s > L \end{cases}.$$

If private saving is impossible, the constant k_s ensures that the IEE (60) holds:

$$\text{(63)} \qquad k_s \;=\; R - \ln E\left[e_s^{\theta_s(a^*+\varepsilon)}\right].$$

If private saving is possible, k_s ensures that the PS constraint (59) holds:

$$\text{(64)} \qquad k_s \;=\; R + \ln E\left[e_s^{-\theta_s(a^*+\varepsilon)}\right].$$

The closed-form solutions allow transparent economic implications. Equation (61) shows that time-t income should be linked to the return not only in period t, but also in all previous periods. Therefore, increases in r_t boost log pay in the current and all future periods equally. Since the CEO is risk averse, it is efficient to spread the reward for good performance over the future to achieve consumption smoothing: the "deferred reward" principle. This result was first derived by Lambert (1983) and Rogerson (1985), who consider a two-period model where the agent only chooses effort.

We now consider how contract sensitivity changes over time. Equation (62) shows that, in an infinite-horizon model ($T \to \infty$), the sensitivity is constant and given by

$$\text{(65)} \qquad \theta_t \;=\; \theta \;=\; (1 - e^{-\delta}) g'(a^*).$$

This is intuitive: the contract must be sufficiently sharp to compensate for the disutility of effort, which is constant. The sensitivity to the current-period return is decreasing in the discount rate—if the CEO is more impatient (higher δ), it is necessary to reward him today more than in the future.

If T is finite, equation (62) shows that θ_t is increasing over time: the "increasing incentives principle." When there are fewer periods over which to spread the reward for effort, the current-period reward ($\partial \mathsf{u}_t / \partial a_t = \theta_t$) must increase to keep the lifetime increase in utility $\partial U / \partial a_t$ constant. Other moral-hazard models predict increasing incentives through different channels. Gibbons and Murphy (1992) generate an increasing current sensitivity because the lifetime increase in utility $\partial U / \partial a_t$ rises over time to offset falling career concerns. In Garrett and Pavan (2015), the current sensitivity rises over time because $\partial U / \partial a_t$ increases to minimize the agent's informational rents. Here, $\partial U / \partial a_t$ is constant since we have no adverse selection or career concerns; instead, the increase in

$\partial \mathsf{u}_t/\partial a_t$ stems from the reduction in consumption smoothing possibilities as the CEO approaches retirement.

While θ_t depends on the model horizon, it is independent of whether private saving is possible—this possibility only affects k_t. Since private saving does not affect the agent's action and thus firm returns, the sensitivity of pay to returns is unchanged. From (61), the possibility of private saving alters the time trend in the level of pay. The log expected growth rate in pay is $\ln E\left[c_t/c_{t-1}\right] = k_t + \ln E\left[e^{\theta_t r_t}\right]$.

If private saving is impossible, substituting for k_t using (63) yields

$$\ln E\left[\frac{c_t}{c_{t-1}}\right] = R - \delta,$$

which is constant over time. If and only if the CEO is more patient than the aggregate economy ($\delta < R$), then the growth rate is positive, as is intuitive. If private saving is possible, (64) yields

$$\ln E\left[\frac{c_t}{c_{t-1}}\right] = R - \delta + \ln E\left[e^{-\theta_t r_t}\right] + \ln E\left[e^{\theta_t r_t}\right].$$

In the limit of small time intervals (or, equivalently, in the limit of small variance of noise σ^2), this yields

$$\ln E\left[\frac{c_t}{c_{t-1}}\right] = R - \delta + \theta_t^2 \sigma_t^2.$$

Thus, the growth rate of consumption is always higher when private saving is possible. This faster upward trend means that the contract effectively saves for the agent, removing the need for him to do so himself. This result is consistent with He (2012), who finds that the optimal contract under private saving involves a wage pattern that is nondecreasing over time.[21] The model thus predicts a positive relationship between pay and tenure, consistent with the common practice of seniority-based pay. Moreover, the growth rate depends on the risk to which the CEO is exposed, which is in turn driven by his incentives θ and firm volatility σ. This is intuitive: greater risk increases the CEO's motive to engage in precautionary saving (since, with CRRA utility, $u'''(c) > 0$), and so a rapidly rising level of pay is necessary to remove the need for him to save privately. Furthermore, in a finite-horizon model, θ_t is increasing over time, so the growth rate of consumption rises with tenure. That is, pay accelerates over time.

To illustrate the economic forces behind the contract, we now present a simple numerical example with $T = 3$, $L = 3$, $\delta = 0$, $a_t^* = a^*$, and $g'(a^*) = 1$. From (62), the contract is

$$\ln c_1 = \frac{r_1}{3} + \kappa_1,$$

$$\ln c_2 = \frac{r_1}{3} + \frac{r_2}{2} + \kappa_2,$$

$$\ln c_3 = \frac{r_1}{3} + \frac{r_2}{2} + \frac{r_3}{1} + \kappa_3,$$

where $\kappa_t = \sum_{s=1}^{t} k_s$. An increase in r_1 leads to a permanent increase in log consumption—it rises by $\frac{r_1}{3}$ in all future periods. In addition, the sensitivity $\partial \mathsf{u}_t/\partial a_t$ increases over time, from 1/3 to 1/2 to 1/1. The total lifetime reward for effort $\partial U_t/\partial a_t$ is a constant 1 in all periods.

[21] Lazear (1979) has a back-loaded wage pattern for incentive, rather than private saving considerations (the agent is risk neutral in his model). Since the agent wishes to ensure he receives the high future payments, he induces effort to avoid being fired. Similarly, in Yang (2010), a back-loaded wage pattern induces agents to work to avoid the firm being shut down.

We now consider $T = 5$, so that the CEO lives after retirement. The contract is now

$$\begin{aligned}(66)\qquad \ln c_1 &= \frac{r_1}{5} + \kappa_1,\\ \ln c_2 &= \frac{r_1}{5} + \frac{r_2}{4} + \kappa_2,\\ \ln c_3 &= \frac{r_1}{5} + \frac{r_2}{4} + \frac{r_3}{3} + \kappa_3,\\ \ln c_4 &= \frac{r_1}{5} + \frac{r_2}{4} + \frac{r_3}{3} + \kappa_4,\\ \ln c_5 &= \frac{r_1}{5} + \frac{r_2}{4} + \frac{r_3}{3} + \kappa_5.\end{aligned}$$

Since the CEO takes no action from $t = 4$, his pay does not depend on r_4 or r_5. However, it depends on r_1, r_2, and r_3 as his earlier efforts affect his wealth, from which he consumes.

Short-Termism. We finally extend the basic model to allow the agent to engage in short-termism, and study how this possibility affects the optimal contract. Short-termism is broadly defined to encompass any action that increases current returns at the expense of future returns—scrapping positive NPV investments (see, for example, Stein 1988) or taking negative NPV projects that generate an immediate return but weaken long-run value (such as subprime lending), earnings management, and accounting manipulation.

At time t, in addition to an effort and savings decision, the manager can also take a myopic action $m_{t,i}$ that increases the current return to $r'_t = r_t + M_i(m_{t,i})$ where M_i is a concave function. The CEO also chooses a "release lag" i, which is the number of periods before the negative consequences of myopia become evident. The maximum possible release lag is $H \leq T - L$. Myopia at t with release lag i reduces the return at $t + i$ to $r'_{t+i} = r_{t+i} - m_{t,i}$, and leaves other returns unchanged ($r'_{t+s} = r_{t+s}$ for $s \neq 0, i$). Let $\mathcal{M}_i = M'_i(0) \in [0, 1)$ be inversely related to the marginal inefficiency of manipulation at release lag i.

If the firm is sufficiently large, the principal will wish to implement zero manipulation, i.e., $m_{t,i} = 0 \; \forall \, t$. If the agent engages in a small myopic action $m_{t,i} \geq 0$ at time t, his utility changes to the leading order by

$$E_t\left[\frac{\partial U}{\partial r_t}\right]\mathcal{M} m_{t,i} - E_t\left[\frac{\partial U}{\partial r_{t+i}}\right] m_{t,i}.$$

This should be weakly negative for all small $m_{t,i} \geq 0$. Hence, we obtain an additional no manipulation (NM) constraint:

$$(67)\qquad \text{NM}: E_t\left[\frac{\partial U}{\partial r_t}\right]\mathcal{M}_i \leq E_t\left[\frac{\partial U}{\partial r_{t+i}}\right]$$

for $t \leq L$. The optimal contract is now as above, with the additional constraint (67).

We apply this constraint to the five-period model of equation (66), with $H = 1$. The optimal contract now changes to

$$\begin{aligned}(68)\qquad \ln c_1 &= \frac{r_1}{5} + \kappa_1,\\ \ln c_2 &= \frac{r_1}{5} + \frac{r_2}{4} + \kappa_2,\\ \ln c_3 &= \frac{r_1}{5} + \frac{r_2}{4} + \frac{r_3}{3} + \kappa_3,\\ \ln c_4 &= \frac{r_1}{5} + \frac{r_2}{4} + \frac{r_3}{3} + \frac{\mathcal{M}_1 r_4}{2} + \kappa_4,\\ \ln c_5 &= \frac{r_1}{5} + \frac{r_2}{4} + \frac{r_3}{3} + \frac{\mathcal{M}_1 r_4}{2} + \kappa_5.\end{aligned}$$

Even though the CEO retires at the end of $t = 3$, his income depends on r_4, otherwise he would have an incentive to boost r_3 at the expense of r_4. Thus, the CEO should retain equity in the firm even after retirement. For a general maximum release lag of H, the CEO should be sensitive to firm returns

until period $L + H$, i.e., retain equity in the firm for H years after retirement. This result formalizes the verbal argument of Bebchuk and Fried (2004), who advocate escrowing the CEO's equity to deter him from inflating the stock price before retirement and then cashing out. For example, Angelo Mozilo, the former CEO of Countrywide Financial, made $129 million from stock sales in the twelve months prior to the start of the subprime crisis. Indeed, in the aftermath of the crisis, banks such as Goldman Sachs and UBS have been increasing vesting horizons. An alternative remedy proposed is to use clawbacks, i.e., pay executives bonuses for good short-term performance, but rescind them if the performance ends up being reversed in the long term. However, clawbacks have been very rare in practice, perhaps because they require an active decision by the board. Lengthening the vesting period of equity so that rewards are not paid out prematurely in the first place may be a superior solution.

The sensitivity to r_4 depends on the efficiency of earnings inflation $\mathcal{M}_1$; in the extreme, if $\mathcal{M}_1 = 0$, myopia is impossible and so there is no need to expose the CEO to returns after retirement. The contract is unchanged for $t \leq 3$, that is, for the periods in which the CEO works. Even under the original contract, there is no incentive to inflate earnings at $t = 1$ or $t = 2$ because there is no discounting, and so the negative effect of myopia on future returns reduces the CEO's lifetime utility by more than the positive effect on current returns increases it. With discounting, incentives increase even faster over time than in the absence of a myopia problem. The higher sensitivity to future returns ensures that myopia causes the CEO to lose enough in the future to counterbalance the effect of discounting.

The contracts in (66) and (68) can be implemented in a simple manner. Each year, the manager's annual pay is escrowed into an "incentive account," a proportion θ_t of which is invested in stock and the remainder in cash, so his %–% incentives equal θ_t given by (62). If the stock price declines, so that the fraction of stock falls below θ_t, cash in the account is used to buy stock to replenish his incentives. Every year, a fraction of the account vests and is paid to the manager, but the remainder remains escrowed to deter myopia. Zhu (2016) shows that "bonus banks," introduced in practice by the consulting firm Stern Stewart, are a similar way to deter myopia. Bonuses for short-term performance are deposited into the "bonus bank," rather than immediately paid to the manager, and only a fraction is paid each period. Poor performance in one period, which may be caused by a myopic action in a previous period, wipes out previously accrued bonuses.

The advantage of the above framework is that it yields closed-form solutions that make the economic intuition transparent. However, it comes at the cost of a number of assumptions. First, as in the EG model, it assumes a fixed target action and that the noise precedes the action, which may be reasonable in some settings but not others. Second, it assumes a fixed retirement date T and does not allow for quits or firings beforehand, which is an important limitation in a CEO setting.

4.2 *Dynamic Incentives in Continuous Time*

This section presents the continuous-time model of Sannikov (2008) which allows for quits and firings, as well as for the implemented effort level to be endogenized. At every instant t, the agent takes action a_t (from a compact set with smallest element 0) and consumes c_t; the framework assumes away private saving so we do not distinguish between income and consumption. His expected lifetime utility at date t is

$$U_t = E_t\left[\int_t^\infty e^{-\delta(s-t)}\mathsf{u}(c_s, a_s)\,ds\right]. \tag{69}$$

It is the "promised utility" that the agent will obtain if he exerts the path of efforts recommended by the principal. This path of efforts is given by an adapted process (a_t^*). Recall that an adapted process is a process whose value depends on the information available at time t.

The principal uses the same discount rate δ, and her expected utility is

$$Q_t = E_t\Big[\int_t^\infty e^{-\delta(s-t)}(dV_s - c_s\,ds)\Big], \tag{70}$$

where dV_t (instantaneous profit before paying the CEO) is assumed to be

$$dV_t = a_t\,dt + \sigma\,dZ_t, \tag{71}$$

where Z_t is a standard Brownian motion.

To derive the optimal contract, we first recall two basic lemmas from stochastic calculus, proven in Appendix A.

Martingale Representation Theorem.—If $U_t = E_t\big[\int_t^\infty e^{-\delta(s-t)}K_s\,ds\big]$ for some adapted process K_t, then

$$dU_t = (-K_t + \delta U_t)\,dt + \xi_t\,dZ_t, \tag{72}$$

where ξ_t is some other adapted process.

Hamilton-Jacobi Bellman (HJB) Equation.—Consider a stochastic process x_t following $dx_t = \mu(x_t, C_t)\,dt + \sigma(x_t, C_t)\,dZ_t$, where C_t is a control variable (which is potentially multidimensional), and the following optimal control problem:

$$Q(x) = \sup_{(C_s)_{s\geq t}} E_t\Big[\int_t^\infty e^{-\delta(s-t)}f(x_s, C_s)\,ds\,|\,x_t = x\Big],$$

where control C_t is adapted, i.e., uses only information available at t, and $f(x_s, C_s)$ is some reward function. Then the value function $Q(\cdot)$ satisfies

$$0 = \sup_C f(x, C) - \delta Q(x) + Q'(x)\mu(x, C) + \tfrac{1}{2}Q''(x)\sigma^2(x, C). \tag{73}$$

Using the martingale representation theorem (72), the agent's utility process (69) can be written

$$dU_t = (-\mathsf{u}(c_t, a_t) + \delta U_t)\,dt + \beta_t\sigma\,dZ_t$$

for some process β_t that represents the sensitivity of the contract (equation (72) yields a process ξ_t, and we set $\beta_t = \xi_t/\sigma$).

This allows us to write the contract. Call a_t^* the time-t action recommended by the principal (it also depends on the past, and will soon be optimized upon). On the equilibrium path, $dV_t = a_t^*\,dt + \sigma\,dZ_t$, and so utility follows:

$$dU_t = (-\mathsf{u}(c_t, a_t^*) + \delta U_t)\,dt + \beta_t(dV_t - a_t^*\,dt). \tag{74}$$

This equation defines an essential part of the contract: given the past (summarized by the promised utility U_t), the contract recommends action a_t^*, and given the "surprise" $dV_t - a_t^*\,dt$, the contract specifies how promised utility changes: by dU_t given in (74). Here (74) features the recommended action a_t^* (which the principal specifies), rather than the actual action a_t (which the principal does not observe).

The intuition is as follows: U_t represents an "account" that contains the lifetime utility promised to the agent. If the principal provides instantaneous utility to the agent, the account changes by $-\mathsf{u}(c_t, a_t^*)$ as less utility is owed in the future. If firm value is higher than expected $(dV_t - a_t^*\,dt > 0)$, the

account rises in value. Otherwise, it grows at a rate δ (the δU_t term).

We now move to the agent's IC. If the agent chooses action a_t, his utility is

$$\max_{a_t} \mathsf{u}(c_t, a_t)\, dt + d\, U_t(a_t),$$

i.e., he receives $\mathsf{u}(c_t, a_t)$ today, and will receive continuation utility of $d\, U_t(a_t)$ later. Given (71) and (74), we have

$$d\, U_t \;=\; \beta_t a_t\, dt + \text{terms independent of } a_t,$$

which yields the IC:

$$a_t^* \;\in\; \arg\max_{a} \mathsf{u}(c_t, a_t) + \beta_t a_t.$$

The first-order condition yields

$$\beta_t \;=\; -\mathsf{u}_a(c_t, a_t^*), \tag{75}$$

i.e., incentives β_t are determined by c_t and a_t^*.

Turning to the principal's problem, the state variable for the HJB stochastic process is $x_t = U_t$, the agent's promised utility. The principal's value function is $Q(x_t)$. Using (75), the HJB equation (73) is, for all x,

$$\begin{aligned} 0 \;=\; & \max_{c,a} a - c - \delta Q(x) + Q'(x) \\ & \times [-\mathsf{u}(c, a) + \delta x] \\ & + \frac{1}{2} Q''(x) \mathsf{u}_a(c, a)^2 \sigma^2. \end{aligned} \tag{76}$$

This gives an ordinary differential equation for $Q(x)$. From this, the optimal c and a are implicitly determined by

$$\begin{aligned} & -1 - Q'(x)\, \mathsf{u}_c(c, a) \\ & \quad + \frac{1}{2} Q''(x)\, \partial_c \big(\mathsf{u}_a(c, a)^2\big)\, \sigma^2 \;=\; 0 \end{aligned} \tag{77}$$

and

$$\begin{aligned} & 1 - Q'(x)\, \mathsf{u}_a(c, a) \\ & \quad + \frac{1}{2} Q''(x)\, \partial_a \big(\mathsf{u}_a(c, a)^2\big)\, \sigma^2 \;=\; 0. \end{aligned} \tag{78}$$

We now need to specify the boundary conditions. The agent's per period reservation utility is w, and so he accepts the contract only if $U_t \leq w/\delta$. Let $Q_0(x) = -\mathsf{u}(\cdot, 0)^{-1} \times (\delta w)/V + A$ denote the cost of providing this to the agent when the agent exerts zero effort, where A is the present value of the principal's outside option, e.g., his surplus from hiring a new agent. The agent is employed if and only if his promised utility is $x \in [x_L, x_H]$, with the following matching and smooth pasting conditions:

$$\begin{aligned} Q(x) \;&=\; Q_0(x), \\ Q'(x) \;&=\; Q_0'(x) \end{aligned} \tag{79}$$

for

$$x \;=\; x_L, x_H.$$

Hence, the problem is characterized by the ODE (76), x_L, and x_H. At x_L, the agent is terminated because of poor performance. At x_H, he is terminated because he has become too expensive to incentivize. Intuitively, as the agent's promised utility increases, his marginal utility of money falls, and so it is harder to incentivize him.

We have four unknowns, x_L, x_H, and the two degrees of freedom associated with a second-order ODE (given a value x_*, the ODE is described by two parameters, $Q(x_*)$ and $Q'(x_*)$). We also have four equations (79). Hence, the problem yields a solution.

To sum up, the principal solves the problem as follows. Given her value function Q, she finds the optimal action and consumption from the first-order conditions (77) and (78). These in turn determine the optimal

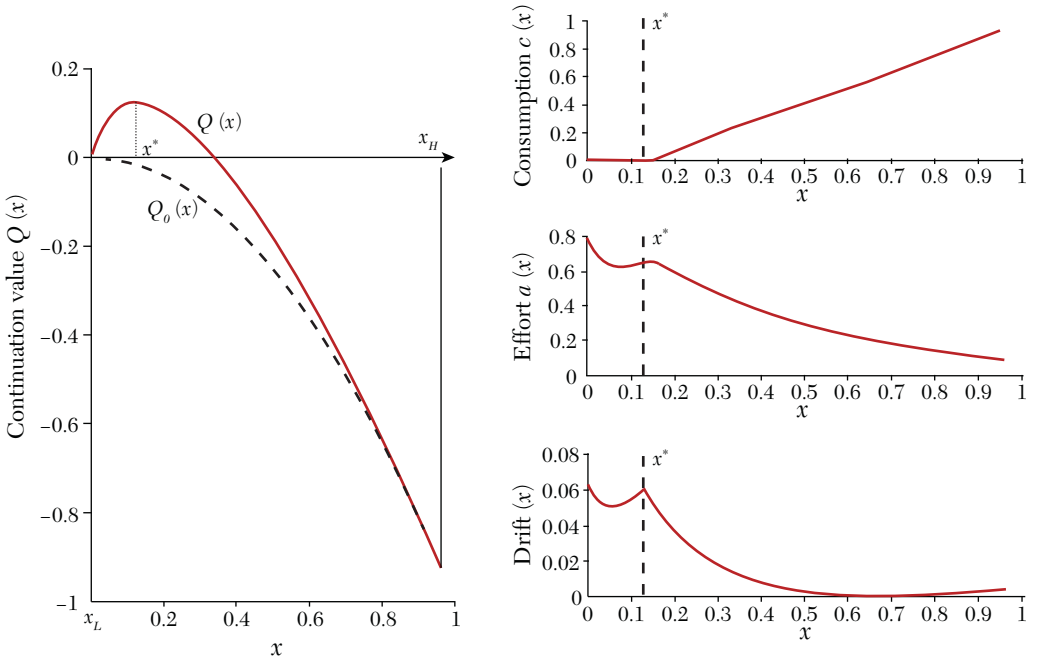

Figure 1. Optimal Contract in Continuous Time

Source: Author replication of Sannikov (2008) with parameters $u(c) = \sqrt{c}$, $g(a) = 0.5a^2 + 0.4a$, $\delta = 0.1$, $\sigma = 1$.

incentives from (75), and the agent's utility is given by (74).

This problem is quite complex, and typically the solutions are numerical.[22] In figure 1, we replicate a result from Sannikov (2008).

While the numerical results depend somewhat on the utility function, for the specification in figure 1, when promised utility rises, consumption increases. As a result, effort falls; the marginal utility of additional consumption is low and so stronger monetary incentives are required to induce effort. Since the agent is now expensive to incentivize, the optimal effort level falls. Other variables have non-monotonic relationships with promised utility. One important open question would be to take these theoretical models to the data and determine which functional form best describes the world. The absence of analytical solutions renders comparative statics relatively difficult to obtain. In many situations, greater risk aversion reduces incentives, for the same intuition as Holmstrom and Milgrom (1987)—incentives are more costly, as they require the agent to be paid a greater risk premium.

Sannikov's (2008) methodology has since been used in executive compensation

[22] One closed-form solution is the one in the Edmans et al. (2012) setup, with a constant a^* (e.g., $a^* = \bar{a}$) and no outside opportunity ($w = -\infty$). With log utility ($u(c, a) = \ln c - g(a)$) and $r = \rho$, the reader can verify that $c(x) = De^{rx}$, $Q(x) = Ae^{rx} + B$ for some constants A, B, D. With CRRA $\left(u = \left(ce^{-g(a)}\right)^{1-\gamma}/(1-\gamma)\right)$, the solution has the form $c = Dx^{1/(1-\gamma)}$, $Q(x) = Ax^{1/(1-\gamma)} + B$. The resulting contracts are described in section 4.1. Otherwise, the only known solutions are numerical.

applications. He (2012) extends the model to allow for private saving. In standard models without private saving (e.g., Rogerson 1985), the optimal wage profile is front loaded, but such a profile will induce the agent to engage in a joint deviation of shirking and saving. He shows that the wage profile is back loaded to deter such private saving. He also finds that pay does not fall upon poor performance, but exhibits a permanent rise after a sufficiently good performance history. This downward rigidity is also predicted by Harris and Holmstrom (1982), but through a quite different channel. Their model features two-sided learning about the agent's ability, rather than moral hazard. Downward rigidity in wages insures the agent against negative news about his ability, while wage raises after positive news ensure that he does not quit.[23]

4.3 *Empirical Analyses*

We now turn to tests of the empirical predictions of dynamic models. Lambert (1983), Rogerson (1985), and Edmans et al. (2012) predict the "deferred reward principle": firm performance should affect future, as well as current, pay due to consumption-smoothing considerations. Indeed, Boschen and Smith (1995) show that firm performance has a much greater effect on the NPV of future pay than current pay. Gibbons and Murphy (1992) find support for the "increasing incentives principle," that incentives rise over time, although studying pay–performance sensitivity rather than wealth–performance sensitivity. This result is consistent with both consumption-smoothing possibilities and career concerns falling with tenure. In addition to incentives, Murphy (1986) finds that pay increases over time, consistent with models that predict a backward-loaded wage pattern to remove incentives for private saving. However, to our knowledge, predictions that the growth rate in pay depends on the level of incentives θ and firm risk σ are as yet untested. Turning to the effects of short-termism, Edmans et al. (2012) predict that firms in which the CEO has greater scope to engage in myopia should have longer vesting periods and also more rapidly increasing incentives over time. Consistent with the first prediction, Gopalan et al. (2014) find that incentives have longer horizons in firms with more growth opportunities and greater R&D intensity.

Turning to the predictions regarding termination, many models predict termination after poor performance, to deter shirking ex ante, e.g., DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007), Biais et al. (2007), and Sannikov (2008).[24] In particular, the first three models feature limited liability, which reduces the principal's ability to punish poor performance financially, thus leading to a role for termination. In some cases, such as Sannikov (2008), termination also arises after very good performance, as the agent becomes too expensive to incentivize. However, historically, dismissals have been relatively rare. Murphy and Zabojnik (2007) report turnover rates of 10 percent per year in the 1970s and 1980s, which increased only to 11 percent in the 1990s. Taylor's (2010) learning model shows that the low rate of dismissals can only be justified if the costs to shareholders of turnover exceed $200 million. Turning to turnover–performance sensitivity, Coughlan and Schmidt (1985), Jensen and Murphy (1990), Warner, Watts,

[23] DeMarzo and Sannikov (2006) use the Sannikov (2008) framework to study optimal capital structure and show that it can be implemented with standard securities—a credit line, long-term debt, and outside equity. Since the agent always holds equity, the model focuses on financing, rather than executive compensation.

[24] Note that termination after poor performance is typically not subgame-perfect, so moral hazard models assume that the firm can commit to terminate the CEO. Learning models predict subgame-perfect termination after poor performance, as such performance signals low managerial quality (e.g., Jovanovic 1979, Taylor 2010, and Garrett and Pavan 2012).

and Wruck (1988), and Weisbach (1988) find that turnover probability is decreasing in performance, but the economic magnitude is small. Jensen and Murphy (1990) estimate that a CEO who performs in line with the market over two years has a 11.1 percent dismissal probability; underperforming by 50 percent in each year increases this probability only to 17.5 percent. Thus, even under the aggressive assumption that the CEO receives no severance package and is unable to find alternative employment until retirement, Jensen and Murphy (1990) estimate that incentives from dismissal are equivalent to an equity stake of 0.03 percent.

Moreover, incentives from dismissal are even lower if the CEO is granted severance pay. In contrast to most theories, which advocate that pay should be weakly monotonic in firm performance, CEOs are often given severance packages upon departure. Yermack (2006) finds a mean contracted severance pay of $0.9 million, with a mean discretionary amount of $4.5 million; the respective maximums are $36.1 and $121.1 million, suggesting that these packages can be substantial. Their usage is especially prevalent among dismissed CEOs, compared to those who voluntarily retire, and thus appears to reward CEOs for failure. However, a closer look at the data suggests that the vast majority of "severance pay" does not stem from compensation for loss of employment, but instead items such as unvested restricted shares, unexercised stock options, and accrued pension benefits, which were promised and contractually obligated to the CEO under any state of nature. For example, out of Henry McKinnell's much-criticized $180 million severance package from Pfizer, $78 million was deferred compensation ($67 million contributed plus $11 million interest), $82 million was the present value of his pension plan, and $8 million was from stock options. Thus, only an incremental $11 million was due to the loss of employment.[25] Furthermore, some theories predict that severance pay can be optimal. Almazan and Suarez (2003) show that it can induce the CEO to leave voluntarily when a more able replacement is available; Inderst and Mueller (2010) demonstrate that it can deter a CEO from entrenching himself by concealing negative information that would lead to his dismissal. One example is to induce the CEO to accept a takeover bid, which typically yields a substantial premium to shareholders. Manso (2011) shows that severance pay is valuable to induce the CEO to explore new technologies, rather than merely exploit existing ones. In the aforementioned model of He (2012), severance pay leads to a backward-loaded wage pattern that is robust to private savings.

Recent studies of CEO turnover have uncovered higher rates. Kaplan and Minton (2012) aggregate both internal (board-driven) and external (through takeover and bankruptcy) turnover and find that total annual turnover was 17.4 percent over 1998–2005. A one standard-deviation fall in the industry-adjusted stock return is associated with a 3.4 percent increase in the likelihood of turnover. This figure is 2.1 percent for industry performance and 1.8 percent for the performance of the overall market. Jenter and Lewellen (2014) find a total turnover rate of 11.8 percent per year, of which they estimate 4.1–4.5 percentage points (35–38 percent of the total) are performance induced, i.e., would not have occurred had performance been good.

Thus, more recent evidence suggests that the threat of job loss from poor performance is significant. While these results support the prediction that firing occurs upon poor performance, they do not support the prediction that it is also prompted by good performance. In addition, moral hazard models typically

[25] We thank David Yermack for this example.

do not yield quantitative predictions for what the rate of firing or the sensitivity of firing to performance should be, making it difficult to assess whether the observed findings are optimal.[26]

5. *Open Questions*

5.1 *Apparent Inefficiencies in Executive Compensation*

Thus far, we have argued that many features of executive compensation that are frequently criticized may yet be consistent with efficient contracting. Examples include the level of pay, low $–$ incentives, the negative relationship between incentives and firm size, the use of stock rather than options, the lack of relative performance evaluation, and the use of severance pay and inside debt. However, this empirical consistency does not prove that compensation practices are efficient. As discussed earlier, the positive correlation between pay and firm size is also consistent with rent extraction, and may arise because a third omitted variable drives both; similar concerns surround other empirical findings consistent with value-maximization models. In addition, even accepting the empirical correlations discussed in this paper as supportive of value maximization in general, there remain several features of compensation that could be improved upon.

First, empirical studies uncover results for the average firm. However, even if compensation is efficient on average, there may still be several individual cases of rent extraction. For example, while the theories discussed in section 4.3 may be able to justify the mean level of severance pay, their forces are unlikely to be strong enough to rationalize extreme realizations, such as the maximum discretionary award of $121.1 million. In addition, while Dittmann, Maug, and Spalt (2013) find that indexation of options would not create value for the average firm, it would for 25 percent of firms, and so the absence of indexation across all firms is difficult to reconcile with efficiency.

Second, some aspects of compensation are hidden from shareholders, which is difficult to reconcile with them being set in shareholders' interest. For example, Lie (2005) presents evidence that the positive stock returns after the disclosed grant dates of executive stock options, first documented by Yermack (1997), arises from backdating. Since options are typically granted at the money, the CEO—unbeknown to shareholders—chooses the grant date in retrospect, to coincide with days on which the stock price is low, thus justifying a low strike price. Similarly, recent corporate scandals such as Tyco uncovered executives extracting perks that were initially unknown to shareholders.

Third, current schemes often fail to keep pace with a firm's changing conditions. While section 3.5 argues that the CEO's incentives are sufficient in normal times to induce effort, if a company encounters difficulties and its stock price falls, the delta of his options declines, and so the options lose much of their incentive effect. One remedy used in practice is the repricing of out-of-the-money options (Brenner, Sundaram, and Yermack 2000; Acharya, John, and Sundaram 2000), but this is controversial, as it appears to reward the CEO for failure. Even if the CEO is paid purely with stock (which always has a delta of one), the problem continues to exist as long as the benefit of effort $b(S)$ is increasing in firm size. Intuitively, when firm size falls, the benefits from effort are lower and so additional equity is needed to induce a given level of effort. For example, if both the production and utility functions are multiplicative, the relevant measure is %–%

[26] Taylor (2010) derives quantitative predictions for the rate of firing as a function of the cost of turnover to shareholders, but in a learning model.

incentives, the percentage of the CEO's wealth that is comprised of stock, and this measure falls when the stock price declines. If the utility function is additive, the relevant measure is \$–% incentives, the dollar value of the CEO's equity, which also falls when the stock price declines. A simple way to replenish incentives is to increase (reduce) the portion of the CEO's salary that is given in equity (cash) after the stock price falls; indeed, Core and Guay (1999) show that firms use new equity grants to move executives toward their optimal incentive levels. Alternatively, the incentive account discussed in section 4.1 involves rebalancing the amount of the CEO's escrowed equity and deferred cash to ensure he always has sufficient equity. Critically, in both cases, unlike the repricing of options, the CEO's additional equity is not given for free: it is paid for by a reduction in cash. Thus, the CEO is reincentivized without him being rewarded for failure.

Fourth, standard measures of CEO incentives, such as those considered in section 3, only measure how the CEO's wealth is affected by changes to the current stock price. They do not consider the extent to which the CEO is also aligned with the long-term stock price, i.e., the horizon of incentives. The classic managerial myopia models of Stein (1988, 1989) show that short-term incentives can lead to myopic actions. In a corporate context, these actions can involve cutting R&D, reducing employee training, writing loans that may become delinquent in the future, or expending corporate resources on earnings management. Empirical studies of the horizon of incentives have been hindered by lack of data availability on the vesting period of an executive's equity, but recent studies suggest that horizons may affect behavior. Gopalan et al. (2014) use the recent change in disclosure requirements to pioneer a measure of the "duration" of incentives, analogous to the duration of a debt security. Shorter duration incentives are correlated with earnings management. Edmans, Fang, and Lewellen (2016) study the quantity of equity scheduled to vest in a given year, since this amount depends on equity grants made several years prior and is thus likely exogenous to current investment opportunities. They find that vesting equity is significantly negatively correlated with the growth rate of R&D and capital expenditure, and positively correlated with the likelihood of meeting or narrowly beating earnings targets. Johnson, Ryan, and Tian (2009) show that unrestricted stock is positively correlated with corporate fraud.

One potential solution to the potential negative consequences of short horizons is to extend the vesting period of equity. While the current debates surround the level of pay and the sensitivity of pay to performance, extending the horizon of incentives may be particularly valuable in overcoming moral hazard. However, lengthening vesting periods is not costless. First, doing so will potentially expose the CEO to risk outside his control. Second, Laux (2012) shows theoretically that, if the CEO forfeits unvested equity upon dismissal, he may engage in myopic actions to avoid the risk of dismissal until his equity has vested. Third, Brisley (2006) demonstrates that unvested equity ties up a significant portion of the CEO's wealth within the firm, and thus may cause him to turn down risky, value-creating projects.

We note that most of the potential remedies—indexation (where valuable), a crackdown on perks, updating contracts, and lengthening vesting periods—can be implemented by shareholders (or shareholder-aligned boards) themselves, without the need for regulatory intervention. The issue with regulation is that it is one-size-fits-all and cannot be adapted to a firm's particular circumstances. For example, a minimum vesting horizon of (say) five years for equity may be too short to induce investment in growth industries, and too long (thus subjecting the CEO to excessive risk) in mature

industries. Indeed, Gopalan et al. (2014) find that equity "duration" is longer in firms with higher growth opportunities, long-term assets, and R&D intensity, suggesting that the optimal vesting period varies according to firm characteristics. The recent increases in disclosure requirements, and say-on-pay legislation, are steps in the direction of allowing shareholders to ensure the optimality of contracts, as both give the information and ability to decide whether a given pay package is appropriate in a particular context.

Moreover, any policy to reform executive pay should not focus narrowly on compensation alone, but recognize the systemic nature of the issue. Bolton, Scheinkman, and Xiong (2006) show that shareholders who wish to maximize the short-term stock price may deliberately induce myopia by voting for short-term CEO contracts. Thus, passing say-on-pay legislation alone may not improve value creation if shareholders do not have the incentives to set pay optimally. Reorienting shareholders to focus on long-term value is critical for ensuring that greater shareholder power does indeed lead to improvements in executive contracts. One potential channel is to encourage shareholders to take large stakes (e.g. through superior liquidity, or fewer disclosure requirements), since large shareholders have sufficient incentives to gather information on the firm's long-run value, rather than relying on public information such as short-term profit (Edmans 2009).

Regulatory intervention is, however, valuable if externalities exist. In Bénabou and Tirole (2016), competition causes firms to offer high incentives to screen for high-ability managers. However, strong incentives also lead to managers focusing excessively on measurable tasks and shirking on unmeasurable tasks, echoing Holmstrom and Milgrom (1991). Acharya and Volpin (2010) and Dicks (2012) show that competition can lead to spillover effects: if one firm overpays its workers (e.g., due to poor corporate governance), this will lead to other firms optimally doing so to remain competitive, even if they are well-governed. How quantitatively important these possible externalities remains an open question.

5.2 *Underexplored Areas*

5.2.1 *Empirical Questions*

We start with potential avenues for future empirical analysis, before turning to ideas for theoretical research. There have been several high-impact empirical studies of executive compensation. Since the debate about the efficiency of pay concerns magnitudes, this is a field in which descriptive statistics alone are illuminating, for example Jensen and Murphy's (1990) and Hall and Liebman's (1998) seminal works on quantifying CEO incentives. Other studies have correlated CEO pay with outcomes such as firm value (e.g., Morck, Shleifer, and Vishny 1988; McConnell and Servaes 1990; and Himmelberg, Hubbard, and Palia 1999). However, assigning causality is very difficult, as there are very few instruments for CEO incentives. Even the very basic question of whether CEO incentives positively affect firm value has not yet been satisfactorily answered. Thus, a first open question is to find good instruments for, or quasi-exogenous shocks to, CEO pay, to allow identification of the effects of incentives. There have been a limited number of attempts in this direction. Palia (2001) argues that CEO experience, education, and age, and firm volatility are instruments for executive compensation. Core and Larcker (2002) study increases in stock ownership mandated by CEOs approaching the minimum levels set by preannounced guidelines. Shue and Townsend (forthcoming b) exploit the fact that options are granted according to multiyear cycles as an instrument for option grants. Edmans,

Fang, and Lewellen (2016) and Edmans et al. (2016) analyze the scheduled vesting of equity resulting from grants made several years prior.

Second, most empirical studies have been focused on public firms in the United States, given the availability of the ExecuComp dataset. Research on compensation practices in private firms would be particularly useful. Since private firms are likely closer to the efficient benchmark, due to the presence of a concentrated shareholder, such data would allow a comparison with similar public firms to assess whether pay in public firms represents rent extraction. For example, Cronqvist and Fahlenbrach (2013) study firms transitioning from public to private ownership. Additional studies investigating private firms in general (in addition to those that were formerly public) would be helpful. Another fruitful direction would be to study international data and analyze the determinants of cross-country differences in CEO pay. Conyon, Core, and Guay (2011) and Fernandes et al. (2013) are useful steps in this direction. Moreover, while data on CEO wealth (an important determinant of both risk aversion and the private benefits from shirking) is typically unavailable in the United States, it is sometimes available in other countries (see, e.g., Becker 2006).

Third, structurally estimating a dynamic moral hazard model may allow us to study questions that are difficult to answer with reduced-form approaches. For example, it may allow us to quantify several important determinants of the optimal contract that are otherwise difficult to measure empirically, such as the CEO's risk aversion, cost of effort, ability to engage in manipulation, and desire for consumption smoothing.[27] Relatedly, it can permit counterfactual analyses such as the effect on firm value of changes in these parameters, or how the possibility of myopia or short-termism changes the contract. In addition, formal joint tests of a theory's quantitative predictions can highlight where the theory fails, thus opening doors to future research.

As examples of structural approaches, Gayle and Miller (2009) study the extent to which moral hazard can explain the rise in CEO pay. Margiotta and Miller (2000), who find that firms would suffer large losses from not contracting optimally, and also estimate the gains that would arise in a first-best world where effort were observable. In addition, managers only require a small risk premium for the risk imposed by incentives—the benefits of incentives substantially outweigh the costs. Gayle and Miller (2015) show that moral-hazard models in which managers can manipulate accounting reports better explain observed contracts than ones in which they cannot, and Gayle, Golan, and Miller (2015) decompose the sources of pay differences between large and small firms. Pan (forthcoming) extends the Gabaix and Landier (2008) assignment model, which matches CEO talent with firm size, to incorporate additional dimensions of heterogeneity—for example, more diversified firms hire CEOs with more cross-industry experience and research-intensive firms hire CEOs who are more prone to innovation—and estimates the importance of match specificity for productivity. Using a learning model, Taylor (2010) estimates the cost of firing that would rationalize observed turnover rates; a similar approach may uncover whether firing rates are optimal from a moral-hazard perspective. Similarly, while existing tests of the rent extraction versus shareholder value hypotheses typically study the cross

[27] Dupuy and Galichon (2014) advance the modeling and econometrics of multidimensional matching models. Techniques such as theirs could be useful in executive compensation.

section, an analysis of time-series dynamics would allow us to study whether the evolution of pay over time is consistent with value maximization.

Fourth, and relatedly, modern dynamic contracting models have generated new empirical predictions that can be tested using a reduced-form approach. Examples include how the level of pay evolves over time and whether this wage growth is increasing in incentives and firm risk, how incentives change over time, and the determinants of the optimal horizon of incentives.

Fifth, while empirical studies have identified a number of determinants of both the level of pay and incentives, there are significant managerial fixed effects in both (Graham, Li, and Qiu 2012; Coles and Li 2013), suggesting that a large component remains unexplained. These fixed effects may result from talent, ability to extract rent, preferences, or other characteristics. In addition, these studies assume separability of the unobserved fixed effect and the other determinants. However, there may be interactions between them; for example, part of the fixed effect may result from talent, and the impact of talent on pay may depend on firm size. Future research can lead to a better understanding of what these unobservable fixed effects may represent, and how their effect may vary with other characteristics already known to affect pay.

5.2.2 *Theoretical Questions*

We now move to open theoretical questions. First, most current market equilibrium models are static. It would be useful to add a dynamic moral hazard problem where incentives can be provided not only through contracts, but also by the threat of firing.[28] This will also allow us to understand what causes CEOs to move between firms. Moreover, the possibility of turnover adds complications to a standard dynamic model of moral hazard. The classic single-firm models of Lambert (1983) and Rogerson (1985) predict that the reduction in CEO pay caused by poor performance should be spread out over all future periods, to optimize risk sharing. However, the CEO may quit if future expected pay is low, reducing consumption smoothing possibilities.[29]

Second, while the "rent extraction" view has been influential, its arguments have been mainly stated verbally, e.g., Bebchuk and Fried (2004). It would be particularly useful to model the rent extraction view and compare its predictions to the data. One example is Kuhnen and Zwiebel (2009), where the manager has freedom to extract perks, but doing so reduces profits and thus shareholders' assessment of the manager's ability, which may lead to him being fired. The model predicts that perk consumption is increasing in production uncertainty (since it is easier to disguise low profits as resulting from a negative shock) and the manager's outside option (since firing is less of a concern). It is decreasing in the uncertainty about the manager's ability, as then profits have a greater effect on shareholders' assessments of his ability and thus their firing decision. They find qualitative support for these predictions, measuring hidden pay with stock options, restricted stock, and annual pay not declared as salary and bonus. A further potential avenue in this line of research would be a rent extraction model that generates quantitative predictions, and allows for a horse race between the two viewpoints.

Third, existing models of CEO pay are single-agent models, but CEOs work in

[28] See Eisfeldt and Kuhnen (2013) for a market equilibrium model with CEO turnover without moral hazard.

[29] The dynamic moral hazard models of DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007), and Biais et al. (2007) assume risk neutrality, and so consumption smoothing is a nonissue.

teams where complementarities between agents exist. As a result, their contracts affect firm value not only directly through affecting the CEO's effort, but also indirectly because the CEO's effort level affects the optimal effort level chosen by workers. This consideration in turn affects the optimal contract for the CEO. Separately, a team setting allows the study of the relative wages of the CEO and other employees, a question that has been of interest to regulators. Edmans, Goldstein, and Zhu (2013) analyze these issues within a CEO setting; Che and Yoo (2001), Kremer (1993), Winter (2004, 2006, 2010), and Gervais and Goldstein (2007) are analyses of contracting under production complementarities in general principal-agent settings.

Fourth, there has been substantial theoretical progress on continuous-time agency models which allow for the contracting problem to be solved with few assumptions. However, the empirical predictions of such models are typically less clear, given the absence of analytical solutions, and because numerical solutions depend on the parameters chosen. Future research may be able to identify clearer implications of these models, in particular comparative statics on how incentives and turnover–performance sensitivity should differ across firms.

Fifth, contracting models assume that the principal and agent decide on the relevant performance measures and a contract at the start of the employment relationship. However, there is evidence that the performance measures may be renegotiated ex post (e.g., Morse, Nanda, and Seru 2011), and that more than half the CEOs of S&P 500 firms do not have an explicit employment contract, instead employing the CEO at will (Gillan, Hartzell, and Parrino 2009). It would be interesting to study the optimal contract when the CEO and firm wait until performance has been realized before negotiating a sharing rule, and under what circumstances an implicit contract can be sustained.

Sixth, with few exceptions, existing executive compensation models are rational. Incorporating behavioral considerations has been successful in other fields of corporate finance (see the survey of Baker and Wurgler 2013) and could be similarly fruitful here. Baker and Wurgler (2013) divide the behavioral corporate finance literature into two fields—managers who are irrational or have nonstandard utility functions, and rational managers exploiting inefficient markets. As an example of the former, Dittmann, Maug, and Spalt (2010) show that incorporating loss aversion can explain the observed mix of stock and options, which standard utility functions cannot. As an example of the latter, Bolton, Scheinkman, and Xiong (2006) show that contracts that emphasize short-term performance may be a rational response to speculative markets. Other potential behavioral phenomena that could be incorporated into compensation models include bounded rationality, overconfidence (overweighting private signals and underweighting public signals), and optimism (overestimating one's own managerial ability or firm quality).

Finally, turning to questions for both theoretical and empirical research, we now have quantitative theories for the level of pay and "demand" side, given the supply of talent. However, we know relatively little on the "supply" side. Given the substantial pay premium that top executives command over other skilled professions (e.g., medicine or law), it would be interesting to study empirically the extent to which this premium results from limited supply and, if so, explore theoretically why supply appears to remain so limited—why more people do not enter the business profession. A related topic is to understand better the nature of the scarcity of CEO talent, e.g., whether it stems from innate skills, experience, lack of

succession planning, and so on.[30] Separately, while learning models (outside the scope of this survey and listed at the end of the introduction) have generally been developed and tested independently of moral hazard ones, theories that combine both learning and moral hazard, or empirical studies that analyze the relative importance of learning versus moral hazard for observed contracts, would be valuable.

6. *Conclusion*

This article has presented a number of value-maximization models of executive compensation under a unifying framework. We commenced with assignment models of the CEO labor market. More talented managers are matched with larger firms, since their talent is scalable. Their talent also allows them to command higher wages, leading to quantitative predictions for the cross-sectional relationship between pay and firm size. Since the dollar benefits of talent are greater in larger firms, the model also implies that pay should rise over time as average firm size grows.

We then moved to static moral hazard models and showed that the correct empirical measure of incentives depends on whether we believe effort has additive or multiplicative effects on firm value and CEO utility. Moreover, if the effect of effort scales with firm size, $–$ incentives should optimally be weaker in smaller firms. If effort is continuous and the optimal effort level is interior and endogenous, as in Holmstrom and Milgrom (1987), incentives should be increasing in the benefits of effort and decreasing in risk, risk aversion, and the cost of effort. In contrast, if effort is binary or continuous and the principal wishes to implement full productive efficiency, risk and risk aversion do not affect incentives, but increase the level of pay. The rarity of relative performance evaluation appears to contradict the Holmstrom (1979) informativeness principle, but we discussed potential rationalizations of this practice. If the principal aims to induce risk taking, as well as effort from the CEO, the contract should be convex and generally contain options, in contrast to the linear contracts advocated by Holmstrom and Milgrom (1987).

We finally discussed dynamic moral-hazard models. To achieve optimal risk sharing, the reward for good performance should be smoothed over future periods. As the CEO approaches retirement, since there are fewer periods over which to engage in smoothing, the sensitivity of current pay to performance should rise. If the CEO can engage in private saving, his wage profile is typically backward loaded, to remove such saving incentives.

While each model has different features and tackles different questions, we highlighted two common themes. First, the modeling assumptions (e.g., whether preferences or production functions are additive or multiplicative, whether actions are continuous or discrete, and whether private saving is possible) can have significant impact on the model's predictions. Second, we emphasized the empirical predictions of the models and compared them to the data. In particular, observed practices that are often interpreted as evidence of rent extraction are also qualitatively, and sometimes quantitatively, consistent with value maximization. However, consistency with observed correlations does not prove that real-life practices are optimal; instead, it merely emphasizes caution before attempting to intervene by regulation. Whether observed contracts result from efficiency or rent extraction is still an open question, and we highlighted other potential avenues for future research. We look forward to seeing this literature continue to evolve.

[30] For the supply of skills of general workers, see Goldin and Katz (2009).

Appendix A: Longer Derivations

A.1 *Proof of Two Core Results on Dynamic Incentives*

Heuristic Proof of the Martingale Representation Theorem (72).—We show two proofs—one rigorous, one heuristic that shows the economic origin of the result.

HEURISTIC PROOF:

We reason by keeping the dt terms, dropping the $O(dt^2)$ terms:

$$\begin{aligned} U_t &= E_t\Big[\int_t^\infty e^{-\delta(s-t)} K_s\, ds\Big] \\ &= E_t\Big[\int_t^{t+dt} e^{-\delta(s-t)} K_s\, ds\Big] \\ &\quad + E_t\Big[\int_{t+dt}^\infty e^{-\delta(s-t)} K_s\, ds\Big] \\ &= K_t\, dt + o(dt) + e^{-\delta dt} \\ &\quad \times E_t\Big[E_{t+dt}\int_{t+dt}^\infty e^{-\delta(s-(t+dt))} K_s\, ds\Big] \\ &= K_t\, dt + o(dt) + e^{-\delta dt} E_t U_{t+dt} \\ &= K_t\, dt + o(dt) \\ &\quad + (1-\delta\, dt)(U_t + E_t dU_t) + o(dt) \\ &= U_t + (K_t - \delta U_t)\, dt \\ &\quad + E_t[dU_t] + o(dt). \end{aligned}$$

Hence,

$$0 = (K_t - \delta U_t)\, dt + E_t[dU_t]. \tag{80}$$

As $dU_t - E_t[dU_t]$ has mean zero, and the only source of randomness is dZ_t, it makes sense that it can be written

$$dU_t - E_t[dU_t] = \xi_t dZ_t \tag{81}$$

for some adapted process ξ_t. Combining this with (80), we obtain (72). ∎

RIGOROUS PROOF.

The following proof (after Sannikov 2008) is more rigorous, but less explicit about the origins of the result. Define

$$V_\infty := \int_0^\infty e^{-\delta s} K_s\, ds$$

and $V_t = E_t[V_\infty]$. Then, V_t is a martingale. That implies that we can write $dV_t = \xi_t e^{-\delta t} dZ_t$ for some adapted process ξ_t. Hence, $V_t = V_0 + \int_0^t \xi_s e^{-\delta s} dZ_s$. On the other hand,

$$\begin{aligned} V_t &= E_t[V_\infty] \\ &= \int_0^t e^{-\delta s} K_s\, ds + E_t \int_t^\infty e^{-\delta s} K_s\, ds \\ &= \int_0^t e^{-\delta s} K_s\, ds + e^{-\delta t} U_t, \end{aligned}$$

as $U_t = E_t \int_t^\infty e^{-\delta(s-t)} K_s\, ds$. Differentiating with respect to t, we obtain

$$\begin{aligned} dV_t &= \xi_t e^{-\delta t} dZ_t \\ &= e^{-\delta t} K_t\, dt + e^{-\delta t} dU_t - \delta e^{-\delta t} U_t\, dt, \end{aligned}$$

hence,

$$dU_t = (\delta U_t - K_t)\, dt + \xi_t dZ_t. \quad ∎$$

Heuristic Proof of the Hamilton-Jacobi Bellman (HJB) Equation (73).—The result is standard, but here we provide a heuristic proof. We first ignore the maximization over C. We have, as in (80),

$$0 = (f(x_t, C_t) - \delta Q_t)\, dt + E_t[dQ_t].$$

Now, by Ito's lemma using $Q_t = Q(x_t)$, $E_t[dQ_t]/dt = Q'(x_t)\,\mu(x_t, C) + \frac{1}{2}\,Q''(x_t) \times \sigma^2(x_t, C)$. Hence,

$$0 = f(x_t, C_t) - \delta Q(x_t) + Q'(x_t)\,\mu(x_t, C_t) + \frac{1}{2}\,Q''(x_t)\,\sigma^2(x_t, C_t).$$

Next, the principal will want to maximize the right-hand side over C_t, hence (73). ∎

A.2 *Proofs of Other Results*

Proof of (42).—Setting the participation constraint (38) to bind, evaluating the expectation on the left-hand side, and equating the exponents yields

$$\phi + \theta E[V] - \frac{1}{2}\,g a^{*2} - \frac{\eta}{2}\,\theta^2\sigma^2 = w.$$

Substituting in (41) yields

$$E[c] = \phi + \theta E[V] = w + \frac{\big(\theta b(S)\big)^2}{2g} + \frac{\eta}{2}\,\theta^2\sigma^2. \tag{82}$$

From (37), the principal's objective function is $E[S + b(S)a^* - c]$. Substituting in (41) and (82) yields an objective function of

$$S + \frac{\theta}{g}\,[b(S)]^2 - w - \frac{(\theta b(S))^2}{2g} - \frac{\eta}{2}\,\theta^2\sigma^2.$$

The first-order condition with respect to θ yields

$$\frac{[b(S)]^2}{g} - \frac{\theta b(S)^2}{g} - \eta\theta\sigma^2 = 0$$

$$\theta = \frac{1}{1 + g\eta\left(\frac{\sigma}{b(S)}\right)^2}.$$

Comparative Statics for Fixed Pay ϕ.—We have

$$\frac{\partial\phi^*}{\partial\eta} = K_\eta\big(3b^4S^4 - b^2gwS^2(\eta\sigma^2 - 2S) + 2g^2\eta\sigma^2S\big),$$

where

$$K_\eta = \frac{b^2\sigma^2S^2}{2\left(b^2S^2 + g\eta\sigma^2\right)^3}.$$

Hence, $\frac{\partial\phi^*}{\partial\eta} > 0$ if and only if

$$3b^4S^4 + 2b^2gS^3 + 2g^2\eta\sigma^2S > b^2gS^2\eta\sigma^2.$$

Further, we have

$$\frac{\partial\phi^*}{\partial\sigma} = K_\sigma\big(3b^4S^4 - b^2gS^2(\eta\sigma^2 - 2S) + 2g^2\eta\sigma^2S\big),$$

where

$$K_\sigma = \frac{b^2\eta\sigma S^2}{\left(b^2S^2 + g\eta\sigma^2\right)^3}.$$

Hence, $\frac{\partial\phi^*}{\partial\sigma} > 0$ if and only if

$$3b^4S^4 + 2b^2gS^3 + 2g^2\eta\sigma^2S > b^2gS^2\eta\sigma^2.$$

We have

$$\frac{\partial\phi^*}{\partial g} = K_g\big(b^6S^6 + 3b^4g\eta\sigma^2S^4 - 2b^2g^2\eta\sigma^2S^2(\eta\sigma^2 - S) + 2g^3\eta^2\sigma^4S\big),$$

where

$$K_g = \frac{b^2 S^2}{2g^2\left(b^2 S^2 + g\eta\sigma^2\right)^3}.$$

Hence, $\frac{\partial \phi^*}{\partial g} > 0$ if and only if

$$b^6 S^6 + 3b^4 g\eta\sigma^2 S^4 + 2b^2 g^2 \eta\sigma^2 S^3 + 2g^3\eta^2\sigma^4 S > 2b^2 g^2 \eta\sigma^2 S^2 \eta\sigma^2. \blacksquare$$

Heuristic Proof of (61)–(64).—See Edmans et al. (2012) for a rigorous proof. We present a heuristic proof in a simple case that conveys the key intuition. We consider $L = T = 2$ and impose the PS constraint. We wish to show that the optimal contract is given by

$$\ln c_1 = g'(a^*)\frac{r_1}{2} + \kappa_1, \tag{83}$$

$$\ln c_2 = g'(a^*)\left(\frac{r_1}{2} + r_2\right) + \kappa_1 + k_2$$

for some constants κ_1 and k_2 that make the participation constraint bind.

Step 1 (Optimal log-linear contract): We first solve the problem in a restricted class where contracts are log-linear, that is,

$$\ln c_1 = \theta_1 r_1 + \kappa_1, \tag{84}$$

$$\ln c_2 = \theta_{21} r_1 + \theta_2 r_2 + \kappa_1 + k_2$$

for some constants $\theta_1, \theta_{21}, \theta_2, \kappa_1, k_2$. This first step is not necessary but clarifies the economics, and in more complex cases is helpful to guess the form of the optimal contract.

First, intuitively, the optimal contract entails consumption smoothing, that is, shocks to consumption are permanent. This observation implies $\theta_{21} = \theta_1$. To prove this, the PS constraint (64) yields

$$1 = E_1\left[\frac{c_1}{c_2}\right] = e^{(\theta_1 - \theta_{21})r_1} E_1\left[e^{-\theta_2 r_2 - k_2}\right]. \tag{85}$$

This must hold for all r_1. Therefore, $\theta_{21} = \theta_1$.

Next, consider total utility U:

$$U = \ln c_1 + \ln c_2 - g(a_1) - g(a_2)$$

$$= 2\theta_1 r_1 + \theta_2 r_2 - g(a_1) - g(a_2) + 2\kappa_1 + k_2.$$

From (58), the two EF conditions are $E_1\left[\frac{\partial U}{\partial r_1}\right] \geq g'(a^*)$ and $E_2\left[\frac{\partial U}{\partial r_2}\right] \geq g'(a^*)$, that is,

$$2\theta_1 \geq g'(a^*) \quad \text{and} \quad \theta_2 \geq g'(a^*).$$

Intuitively (and as can be proven), the EF constraints should bind, else the CEO is exposed to unnecessary risk. Combining the binding version of these constraints with (84) yields (83).

Step 2 (Optimality of log-linear contracts): We next verify that optimal contracts should be log-linear. Equation (58) yields $d(\ln c_2)/dr_2 \geq g'(a^*)$. The cheapest contract involves this local EF condition binding, that is,

$$d(\ln c_2)/dr_2 = g'(a^*) \equiv \theta_2. \tag{86}$$

Integrating yields the contract

$$\ln c_2 = \theta_2 r_2 + B(r_1), \tag{87}$$

where $B(r_1)$ is a function of r_1, which we will determine shortly. It is the integration "constant" of equation (86) viewed from time 2.

We next apply the PS constraint (64) for $t = 1$:

$$1 = E_1\left[\frac{c_1}{c_2}\right] = E_1\left[\frac{c_1}{e^{\theta_2 r_2 + B(r_1)}}\right] = E_1\left[e^{-\theta_2 r_2}\right] c_1 e^{-B(r_1)}. \tag{88}$$

Hence, we obtain

$$\ln c_1 = B(r_1) + K', \tag{89}$$

where the constant K' is independent of r_1. (In this proof, K', K'', and K''' are constants independent of r_1 and r_2.) Total utility is

$$U = \ln c_1 + \ln c_2 + K'' \tag{90}$$
$$= \theta_2 r_2 + 2B(r_1) + 2K' + K''.$$

We next apply (58) to (90) to yield $2B'(r_1) \geq g'(a^*)$. Again, the cheapest contract involves this condition binding, that is, $2B'(r_1) = g'(a^*)$. Integrating yields

$$B(r_1) = g'(a^*)\frac{r_1}{2} + K'''. \tag{91}$$

Combining (91) with (89) yields: $\ln c_1 = g'(a^*)\frac{r_1}{2} + \kappa_1$, for another constant κ_1. Combining (91) with (87) yields:

$$\ln c_2 = g'(a^*)\left(\frac{r_1}{2} + r_2\right) + \kappa_1 + k_2,$$

for some constant k_2.

Appendix B: Further Detail on Holmstrom–Milgrom (1987)

This section provides further details on the role played by exponential utility, a pecuniary cost of effort, and normal noise in the Holmstrom and Milgrom (1987) model. In the general objective function (10), we now assume $u(x) = x$ so that the cost of effort is non-pecuniary, and have a general (rather than exponential) function $v(\cdot)$. We assume the contract can be rewritten $c(V) = \phi + j(V)$ where ϕ is the fixed component of the contract and $j(V)$ is a possibly non-linear function; this is without loss of generality since ϕ can be zero. The agent's first-order condition becomes

$$\mathrm{E}\left[v'\left(\phi + j(S + b(S)\,a + \varepsilon)\right)j'(V)\,b(S)\right] = g'(a).$$

The agent's reservation utility w affects the fixed salary ϕ, which in turn has two effects on his effort choice. First, it affects his benefit from effort. A higher ϕ reduces the marginal utility of money $v'\left(\phi + j(V)\right)$ because the agent is risk averse. However, it does not affect the marginal cost of effort, because effort entails disutility rather than a financial expenditure. Thus, with a linear contract, the optimal action will depend on ϕ. Second, it affects the agent's attitude towards risk ε. The noise realization affects the agent's benefit from effort since he is risk averse. For example, if noise turns out to be high, then the agent will be highly paid even with low effort; thus, the benefits from working are lower: $v'(\cdot)$ falls. Hence, the agent will integrate over the possible noise realizations when making his effort choice. Since ϕ also lies in the marginal felicity function $v'(\cdot)$, it affects the agent's attitude towards risk and thus his effort choice.

To remove the first effect, HM assume a pecuniary cost of effort, which corresponds to $v(c) = c$ and a general $u(\cdot)$ in the objective function (10). Thus, the first-order condition becomes

$$E\left[u'(\phi + j(S + b(S)\,a + \varepsilon)) \times \left(j'(V)\,b(S) - g'(a)\right)\right] = 0.$$

Now, the marginal benefit of effort $j'(V)\,b(S)$ and the marginal cost of effort $g'(a)$ are on the same footing: both lie inside the final term in parentheses. A high fixed wage ϕ reduces the benefit of effort, but also the cost of effort because this cost is in financial terms. However, ϕ still affects the attitude to risk since it is inside the $u'(\cdot)$ term. Thus, to remove this second effect, we also need exponential utility, $u(x) = -e^{-\eta x}$, so that the objective function (10) becomes

$$E\left[-e^{-\eta(\phi + j(V) - g(a))}\right] = e^{-\eta\phi}E\left[-e^{-\eta(j(V) - g(a))}\right]$$

with first-order condition

(92)

$$E\Big[\eta e^{-\eta(j(S+b(S)a+\varepsilon)-g(a))} \times (j'(S+b(S)a+\varepsilon)b(S)-g'(a))\Big] = 0.$$

The fixed salary ϕ, and thus the reservation utility w, is now irrelevant. However, the contract is still very difficult to solve as we cannot factor out the noise ε. Again, since the incentive constraint (92) must hold only on average, there are many possible contracts that will implement a given action a^*. The contract will depend critically on the distribution of noise ε, which poses important practical challenges as the noise distribution is often unknown. Only with normal noise are we able to calculate the expectation, since then $\mathrm{E}\big[e^{-\eta\varepsilon}\big] = \frac{1}{2}\eta^2\sigma^2$.

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