Internet Appendix for "Does Improved Information Improve Incentives?"

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B Separate signal and output

In the core model, output q is the only contractible variable. As discussed in Section 2, the model remains essentially unchanged if output were unobservable and contracts were instead based on a separate signal s. This section considers an alternative model in which both output q and a separate signal s are contractible, and changes in precision affect the distribution of s rather than output q. This setting allows the model to apply to improvements in monitoring technology when the agent's output is also contractible.

In our core model, there was a single contractible signal (output) and so we could consider general signal distributions. With two contractible signals, tractability requires us to specialize to a binary effort and signal distribution.[21]

Let (q, s) denote a state, which consists of an output $q \in \mathbb{R}$ and a signal $s \in \{L, H\}$. Let $f_\theta(q, s|e)$ denote the PDF of state (q, s) conditional on effort e and precision θ. As in Section 3.3.2, we assume binary effort ($\mathcal{E} = \{\underline{e}, \bar{e}\}$) where the principal wishes to implement $\bar{e}$. Let $p_\theta(e) \equiv \Pr(s = H|e, \theta)$ denote the probability of a high signal, which is good news about effort: $p_\theta(\bar{e}) \geq p_\theta(\underline{e})$.[22] Define the likelihood ratio $I(\theta) \equiv \frac{p_\theta(\bar{e})}{p_\theta(\underline{e})} \geq 1$. Now, the precision parameter θ orders the likelihood ratio, i.e., $I'(\theta) \geq 0$. This condition can be rewritten:

$$\frac{\frac{\partial p_\theta}{\partial \theta}(\bar{e})}{p_\theta(\bar{e})} \geq \frac{\frac{\partial p_\theta}{\partial \theta}(\underline{e})}{p_\theta(\underline{e})}. \tag{46}$$

We first show that the optimal contract is an option, due to the same intuition as before, but now the strike price $X_{s,\theta}$ decreases in the signal realization (as in Chaigneau, Edmans, and Gottlieb (2016)). Since a high signal is good news about effort, rewarding the agent with a lower strike price improves incentives.

[21] This is not possible in the core model where signal is output, since we need more than two outputs to study the structure of the optimal contract (with only two outputs, the contract would involve only two payments, and so options would be indistinguishable from stock or bonuses). Innes (1993) considers two contractible signals, both with general signal distributions. However, the second signal is not informative about effort, which is why a general distribution is feasible without a loss of tractability.

[22] This assumption is without loss of generality, since we can always relabel states.

Lemma 5 *There exists an optimal contract with* $W_\theta(q,s) = \max\{0, q - X_{s,\theta}\}$*, where* $X_{H,\theta} < X_{L,\theta}$.

In the core model, where precision affects output, it affects incentives by changing the likelihood that output exceeds the strike price, and thus that the agent benefits from marginal increases in output. In this extension, precision affects the signal but not output, and so this effect disappears. Instead, precision affects incentives by having differential effects on the likelihood that working and shirking lead to a high signal and thus low strike price. This effect is given in Proposition 5, which is the analogy of Proposition 1 in the main model:

Proposition 5 *(Effect of precision on incentives, separate signal) The incentive effect of precision is positive (negative) if*

$$\frac{\partial p_\theta(\bar{e})}{\partial\theta}\left[X_{L,\theta} - X_{H,\theta} - \int_{X_{H,\theta}}^{X_{L,\theta}} F(q|\bar{e})dq\right] - \frac{\partial p_\theta(\underline{e})}{\partial\theta}\left[X_{L,\theta} - X_{H,\theta} - \int_{X_{H,\theta}}^{X_{L,\theta}} F(q|\underline{e})dq\right] \geq (\leq) 0. \tag{47}$$

The incentive effect comprises two components. The first is how precision increases the probability that working leads to a high signal (and thus low strike price), $\frac{\partial p_\theta(\bar{e})}{\partial\theta}$, multiplied by the increase in the agent's expected wage upon a low strike price. If the agent always exercised his call option, he would always pay the strike price, and so he would benefit fully by the lower strike price $X_{L,\theta} - X_{H,\theta}$. However, since he does not always exercise the option and pay the strike price, his benefit is reduced by $\int_{X_{H,\theta}}^{X_{L,\theta}} F(q|\bar{e})dq$.[23] The overall effect is positive: since the value of the agent's call option is decreasing in the strike price, (10) yields $\frac{d}{dX}\left[X - \int_{-\infty}^{X} F(q|e)dq\right] \geq 0$. The second is how precision increases the probability that shirking leads to a high signal, multiplied by the increase in the agent's expected wage upon a high signal.

[23] To further understand the origin of this term, note that the value of a put option is given by

$$\Pr(q < X|e)\,\mathbb{E}\left[(X-q)\,|q < X, e\right] = \int_{-\infty}^{X} -(q-X)\,f(q|e)dq = \int_{-\infty}^{X} F_\theta(q|e)\,dq,$$

where the final equality follows from integration by parts. Thus, $\int_{X_{H,\theta}}^{X_{L,\theta}} F(q|\bar{e})dq = \int_{-\infty}^{X_{L,\theta}} F(q|\bar{e})dq - \int_{-\infty}^{X_{H,\theta}} F(q|\bar{e})dq$ is the difference in the value of two put options. Indeed, equation (10) can be interpreted as the familar put-call parity equation. The agent's call option contains an implicit put option – the option not to exercise the call, and thus not to pay the strike price. Thus, the agent's gain from a lower strike price is reduced by the fall in the value of his implicit put option.

From (46), the effect of precision on the probabilities is very general: the proportional increase in the probability of a high signal upon working exceeds the proportional increase in the probability of a high signal upon shirking, but it could be that both probabilities increase, both decrease, or the former increases and the latter decreases. Due to this generality, the incentive effect of precision can be positive or negative. The examples below respectively give situations in which θ increases incentives, decrease incentives, and has no effect on incentives.

Example 3 *Suppose* $\frac{\partial p_\theta}{\partial \theta}(\underline{e}) \leq 0$ *and* $\frac{\partial p_\theta}{\partial \theta}(\bar{e}) \geq 0$ *for all* θ*. Then, from (47),* θ *increases incentives.*

If precision increases the probability that working leads to a high signal and reduces the probability that shirking leads to a high signal, incentives automatically rise.

Example 4 *Let* $p_\theta(\bar{e}) = 1 - \theta$ *and let* $p_\theta(\underline{e}) = \frac{1}{2} - \theta$ *for* $\theta \in (0, \frac{1}{2})$*. Then, the LHS of (47) is equal to*

$$\int_{X_{H,\theta}}^{X_{L,\theta}} [F(q|\bar{e}) - F(q|\underline{e})]\, dq < 0,$$

where the inequality follows from FOSD, and so precision decreases incentives.

Here, a rise in θ *reduces* the probability that both working and shirking lead to a high signal by the same absolute amount. However, because θ involves a lower proportional reduction in $p_\theta(\bar{e})$, the likelihood ratio still rises and so a rise in θ still corresponds to greater precision. Consider the limit case where $\theta \to \frac{1}{2}$. Then, $p_\theta(\bar{e}) \to \frac{1}{2}$ and $p_\theta(\underline{e}) \to 0$, so the likelihood ratio tends to infinity: a high signal is perfectly informative. A fall in θ reduces precision because the high signal could now be generated by shirking. However, it *increases* the agent's incentives. The probabilities that both working and shirking generate the high signal – and thus the low strike price – increase by the same amount. Under working, the agent benefits more from the lower strike price, because he is more likely to exercise the call option and pay the lower strike price.

Example 5 *Suppose that signals are entirely uninformative if precision is sufficiently low. That is, there exists* $0 \in \Theta$ *under which* $p_0(\bar{e}) = p_0(\underline{e})$*. Then, we must have* $X_{H,0} = X_{L,0}$ *and*

$$\left.\frac{\partial \Psi}{\partial \theta}\right|_{\theta=0} = \frac{\partial p_\theta}{\partial \theta}(\bar{e}) \times 0 - \frac{\partial p_\theta}{\partial \theta}(\underline{e}) \times 0 = 0.$$

If the initial signal is entirely uninformative, an increase in precision has no incentive effect.

When the signal is entirely uninformative, the strike price is optimally independent of the signal realization. Then, improvements in signal precision do not directly affect the value of the option, so the incentive effect is zero.

B.1 Proofs

Proof of Lemma 5

Let $(W^*(q,s), e^*)$ be a feasible contract and consider an option contract $W_s^O = \max\{0, q - X_s\}$ where the strike price X_s is chosen so that, for each realization of the signal s, both contracts have the same expected payment under effort e^*:

$$\int_{-\infty}^{\infty} W^*(q,s) f(q|e^*) dq = \int_{-\infty}^{\infty} W^O(q,s) f(q|e^*) dq.$$

It is straightforward to show that the option contract W^O exists and is unique. As in the proof of Lemma 1, we will verify that replacing W^* by W^O increases effort, which, in turn, raises the principal's expected profit.

Let

$$e^O \in \arg\max_{e \in \mathcal{E}} \sum_{s=H,L} \int_{-\infty}^{\infty} W^O(q,s) f_\theta(s,q|e) dq - C(e).$$

Since the agent chooses effort e^* when offered W^* and e^O when offered W^O, we must have:

$$\begin{aligned} \sum_{s=H,L} \int_{-\infty}^{\infty} W^O(q,s) f_\theta(q,s|e^O) dq - C(e^O) &\geq \sum_{s=H,L} \int_{-\infty}^{\infty} W^O(q,s) f_\theta(q,s|e^*) dq - C(e^*), \\ \sum_{s=H,L} \int_{-\infty}^{\infty} W^*(q,s) f_\theta(q,s|e^*) dq - C(e^*) &\geq \sum_{s=H,L} \int_{-\infty}^{\infty} W^*(q,s) f_\theta(q,s|e^O) dq - C(e^O). \end{aligned}$$

Combining these two inequalities, we obtain

$$\sum_{s=H,L} \int_{-\infty}^{\infty} \left[W^O(q,s) - W^*(q,s)\right] \left[f_\theta(q,s|e^O) - f_\theta(q,s|e^*)\right] dq \geq 0 \tag{48}$$

Since, conditional on each s, both contracts have the same expected value under effort

e^* and the option contract pays the lowest feasible amount for $q < X_s$ and has the highest possible slope for $q > X_s$, there exists $\bar{q}_s \geq X_s$ such that

$$W^O(q,s) \left\{ \begin{array}{c} \leq \\ \geq \end{array} \right\} W^*(q,s) \text{ for all } q \left\{ \begin{array}{c} \leq \\ \geq \end{array} \right\} \bar{q}_s. \tag{49}$$

In order to obtain a contradiction, suppose that $e^* > e^O$. By MLRP, $\frac{f_\theta(q_H,s|e^O)}{f_\theta(q_H,s|e^*)} \leq \frac{f_\theta(q_L,s|e^O)}{f_\theta(q_L,s|e^*)}$ for any $q_H \geq q_L$ and any s. Rewrite (48) as

$$0 \leq \sum_{s=H,L} \int_{-\infty}^{\infty} \left[W^O(q,s) - W^*(q,s)\right] \left[\frac{f_\theta(q,s|e^O)}{f_\theta(q,s|e^*)} - 1\right] f_\theta(q,s|e^*)dq$$

$$= \sum_{s=H,L} \int_{-\infty}^{\infty} \left[W^O(q,s) - W^*(q,s)\right] \left[\frac{f_\theta(q,s|e^O)}{f_\theta(q,s|e^*)} - 1\right] f_\theta(q,s|e^*)dq + \sum_{s=H,L} \underbrace{\int_{-\infty}^{\infty} \left[W^O(q,s) - W^*(q,s)\right] f_\theta(q,s|e^*)dq}_{0}$$

$$= \sum_{s=H,L} \int_{-\infty}^{\infty} \left[W^O(q,s) - W^*(q,s)\right] \frac{f_\theta(q,s|e^O)}{f_\theta(q,s|e^*)} f_\theta(q,s|e^*)dq$$

$$= \sum_{s=H,L} \left\{ \begin{array}{c} \int_{-\infty}^{\bar{q}_s} \left[W^O(q,s) - W^*(q,s)\right] \frac{f_\theta(q,s|e^O)}{f_\theta(q,s|e^*)} f_\theta(q,s|e^*)dq \\ + \int_{\bar{q}_s}^{\infty} \left[W^O(q,s) - W^*(q,s)\right] \frac{f_\theta(q,s|e^O)}{f_\theta(q,s|e^*)} f_\theta(q,s|e^*)dq \end{array} \right\}$$

$$< \sum_{s=H,L} \frac{f_\theta(\bar{q}_s,s|e^O)}{f_\theta(\bar{q}_s,s|e^*)} \left\{ \begin{array}{c} \int_{-\infty}^{\bar{q}_s} \left[W^O(q,s) - W^*(q,s)\right] f_\theta(q,s|e^*)dq \\ + \int_{\bar{q}_s}^{\infty} \left[W^O(q,s) - W^*(q,s)\right] f_\theta(q,s|e^*)dq \end{array} \right\}$$

$$= \sum_{s=H,L} \frac{f_\theta(\bar{q}_s,s|e^O)}{f_\theta(\bar{q}_s,s|e^*)} \left\{ \int_{-\infty}^{\infty} \left[W^O(q,s) - W^*(q,s)\right] f_\theta(q,s|e^*)dq \right\} = 0$$

where the last inequality follows by the definition of W^O. But this is a contradiction ($0 < 0$), so we must have $e^* \leq e^O$.

Next, we show that the principal's profit is higher with the option contract:

$$\sum_{s=H,L} \int_{-\infty}^{\infty} \left[q - W^O(q,s)\right] f_\theta\left(q,s|e^O\right) dq \geq \sum_{s=H,L} \int_{-\infty}^{\infty} \left[q - W^*(q,s)\right] f_\theta(q,s|e^*)\, dq$$

Subtracting $\sum_{s=H,L} \int_{-\infty}^{\infty} \left[q - W^O(q,s)\right] f_\theta(q,s|e^*)\, dq$ from both sides and rearranging, gives:

$$\sum_{s=H,L} \int_{-\infty}^{\infty} \left[q - W^O(q,s)\right] f_\theta\left(q, s | e^O\right) dq \geq \sum_{s=H,L} \int_{-\infty}^{\infty} \left[q - W^O(q,s)\right] f_\theta\left(q, s | e^*\right) dq,$$

showing that the principal indeed profits from the substitution.

It remains to be shown that $X_H < X_L$. Since the optimal contract is an option, we can write the principal's program as

$$\min_{X_L, X_H} \; p_\theta(\bar{e}) \int_{X_H}^{\infty} (q - X_H) f(q|\bar{e}) dq + [1 - p_\theta(\bar{e})] \int_{X_L}^{\infty} (q - X_L) f(q|\bar{e}) dq$$

subject to

$$\int_{X_H}^{\infty} (q - X_H) \left[f(q|\bar{e}) p_\theta(\bar{e}) - f(q|\underline{e}) p_\theta(\underline{e})\right] dq + \int_{X_L}^{\infty} (q - X_L) \left\{f(q|\bar{e}) \left[1 - p_\theta(\bar{e})\right] - f(q|\underline{e}) \left[1 - p_\theta(\underline{e})\right]\right\} dq \geq C$$

The necessary first-order conditions give:

$$-p_\theta(\bar{e}) \left[1 - F(X_H|\bar{e})\right] + \lambda \left\{p_\theta(\bar{e}) \left[1 - F(X_H|\bar{e})\right] - \left[1 - F(X_H|\underline{e})\right] p_\theta(\underline{e})\right\} = 0$$

$$\therefore \frac{1}{\lambda} = 1 - \frac{p_\theta(\underline{e}) \left[1 - F(X_H|\underline{e})\right]}{p_\theta(\bar{e}) \left[1 - F(X_H|\bar{e})\right]}.$$

$$-\left[1 - p_\theta(\bar{e})\right] \left[1 - F(X_L|\bar{e})\right] + \lambda \left\{\left[1 - p_\theta(\bar{e})\right] \left[1 - F(X_L|\bar{e})\right] - \left[1 - p_\theta(\underline{e})\right] \left[1 - F(X_L|\underline{e})\right]\right\} = 0$$

$$\therefore \frac{1}{\lambda} = 1 - \frac{\left[1 - p_\theta(\underline{e})\right] \left[1 - F(X_L|\underline{e})\right]}{\left[1 - p_\theta(\bar{e})\right] \left[1 - F(X_L|\bar{e})\right]}.$$

Thus, the optimality conditions are

$$\frac{1 - p_\theta(\bar{e})}{p_\theta(\bar{e})} \cdot \frac{1 - F(X_L|\bar{e})}{1 - F(X_H|\bar{e})} = \frac{1 - p_\theta(\underline{e})}{p_\theta(\underline{e})} \cdot \frac{1 - F(X_L|\underline{e})}{1 - F(X_H|\underline{e})}.$$

Note that $p_\theta(\bar{e}) > p_\theta(\underline{e})$ implies $\frac{1-p_\theta(\bar{e})}{p_\theta(\bar{e})} < \frac{1-p_\theta(\underline{e})}{p_\theta(\underline{e})}$. Thus, the optimality condition implies

$$\frac{1 - F(X_L|\bar{e})}{1 - F(X_L|\underline{e})} > \frac{1 - F(X_H|\bar{e})}{1 - F(X_H|\underline{e})}.$$

We claim that this implies that $X_H < X_L$. To see this, note that differentiation gives

$$\frac{d}{dX}\left(\frac{1-F(X|\bar{e})}{1-F(X|\underline{e})}\right) = \frac{1-F(X|\bar{e})}{1-F(X|\underline{e})}\left[\frac{f(X|\underline{e})}{1-F(X|\underline{e})} - \frac{f(X|\bar{e})}{1-F(X|\bar{e})}\right] > 0,$$

which is negative because strict MLRP implies:

$$\frac{f(X|\bar{e})}{1-F(X|\bar{e})} < \frac{f(X|\underline{e})}{1-F(X|\underline{e})}.$$

Indeed, since $\frac{f(q|\bar{e})}{f(q|\underline{e})}$ is strictly increasing in q by MLRP, $\frac{f(q+\delta|\bar{e})}{f(q+\delta|\underline{e})} > \frac{f(q|\bar{e})}{f(q|\underline{e})}$ for all $\delta > 0$. Rearrange this inequality as $f(q|\underline{e})f(q+\delta|\bar{e}) > f(q+\delta|\underline{e})f(q|\bar{e})$. Integrate both sides with respect to δ:

$$f(q|\underline{e})\underbrace{\int_0^\infty f(q+\delta|\bar{e})d\delta}_{1-F(q|\bar{e})} > f(q|\bar{e})\underbrace{\int_0^\infty f(q+\delta|\underline{e})d\delta}_{1-F(q|\underline{e})} \;\Leftrightarrow\; \frac{f(q|\underline{e})}{1-F(q|\underline{e})} > \frac{f(q|\bar{e})}{1-F(q|\bar{e})}.$$

Proof of Proposition 5

Let Ψ denote the agent's marginal benefit from effort. Using the decomposition of the expected wage in equation (10), this can be rewritten

$$\begin{aligned}\Psi(\theta; X_H, X_L) \equiv\; & \mathbb{E}[q|\bar{e}] - \mathbb{E}[q|\underline{e}] \\ & + p_\theta(\underline{e})\left[X_H - \int_{-\infty}^{X_H} F(q|\underline{e})dq\right] + (1-p_\theta(\underline{e}))\left[X_L - \int_{-\infty}^{X_L} F(q|\underline{e})dq\right] \\ & - p_\theta(\bar{e})\left[X_H - \int_{-\infty}^{X_H} F(q|\bar{e})dq\right] - (1-p_\theta(\bar{e}))\left[X_L - \int_{-\infty}^{X_L} F(q|\bar{e})dq\right].\end{aligned}$$

The agent's incentive constraint requires his marginal benefit from effort to exceed its cost, i.e.,

$$\Psi(\theta; X_H, X_L) \geq C. \tag{50}$$

As in Section 3.3.2, precision increases (decreases) incentives if it relaxes (tightens) the incentive constraint (50), i.e., $\frac{\partial \Psi}{\partial \theta}(\theta; X_{H,\theta}, X_{L,\theta}) \geq (\leq) 0$. Differentiating Ψ yields Proposition 5.

C Risk aversion

We now extend the core model to incorporate risk aversion. We make the following small changes to the model to allow us to use the framework of Jewitt, Kadan, and Swinkels (2008) which solves for optimal contracts with contracting constraints (but do not study effect of changes in precision); all other assumptions are unchanged.

Output is now given by $q \in [\underline{q}, \overline{q}]$ and the agent chooses effort in $e \in \left(0, \overline{E}\right)$. The main change is that the agent's utility of wealth is now given by $u(W)$, which is twice continuously differentiable with $u' > 0$, $u'' < 0$, and $\lim_{W \to \infty} u(W) = \infty$. His objective function is $\mathbb{E}[u(W_\theta(q))|e] - C(e)$, with $C > 0, C' > 0, C'' > 0$.

We continue to assume limited liability and that the participation constraint is slack. However, we no longer need to assume the monotonicity constraint. With risk neutrality, Innes (1990) showed that, without monotonicity, the optimal contract is highly discontinuous: the agent receives 0 for $q < \widetilde{X}$, and the entire output q (rather than the residual $q - X$) for $q \geq \widetilde{X}$. Thus, the agent's wage jumps from 0 to q at $q = \widetilde{X}$. Thus, monotonicity is required to rule out discontinuous contracts. With risk aversion, monotonicity is not necessary for the contract to be continuous.

The principal wishes to implement effort level e^*. As in Section 3.2, we assume that the FOA is valid and that output has a location parameter, and we also assume that an optimal contract exists. Thus, for a given θ, the principal's problem is to choose a function $W_\theta(\cdot)$ to minimize $\mathbb{E}[W_\theta(q)|e^*]$, subject to limited liability and the following incentive constraint (for simplicity, we drop the subscript θ from the PDF):

$$\frac{d}{de}\int_{-\infty}^{\infty} u(W_\theta(q)) f(q|e^*) dq = C'(e^*). \tag{51}$$

We refer to the LHS of (51) as "effort incentives". Proposition 1 in Jewitt, Kadan, and Swinkels (2008) implies that the optimal contract is defined implicitly by:

$$\frac{1}{u'(W_\theta(q))} = \begin{cases} \mu\frac{f_e(q|e^*)}{f(q|e^*)} & \text{if } \mu\frac{f_e(q|e^*)}{f(q|e^*)} \geq \frac{1}{u'(0)}, \\ \frac{1}{u'(0)} & \text{if } \mu\frac{f_e(q|e^*)}{f(q|e^*)} < \frac{1}{u'(0)}, \end{cases} \tag{52}$$

where $\mu > 0$, the shadow price of the incentive constraint, is unique, and where $f_e(q|e)$ denotes the first derivative of the PDF with respect to e. Let q^* be implicitly defined by $\mu\frac{f_e(q^*|e^*)}{f(q^*|e^*)} = \frac{1}{u'(0)}$ which is unique due to MLRP. Intuitively, q^* is the highest value of q such that the wage is zero under the optimal contract. It is analogous to the threshold X_θ in Lemma 1, except that it is not called a strike price, since the optimal contract need not be an option.

With MLRP, (52) can be rewritten as

$$W_\theta(q) = \begin{cases} u'^{-1}\left(1/\left(\mu\frac{f_e(q|e^*)}{f(q|e^*)}\right)\right) & \text{if } q \geq q^*, \\ 0 & \text{if } q < q^*. \end{cases} \tag{53}$$

Let $g_\theta(x) \equiv f(x|0)$.[24] We can rewrite:

$$\int_{-\infty}^{\infty} u(W_\theta(q))f(q|e^*)dq = \int_{-\infty}^{\infty} u(W_\theta(e^*+\varepsilon))g_\theta(\varepsilon)d\varepsilon,$$

and so the incentive constraint in (51) becomes

$$\int_{-\infty}^{\infty} W_\theta'(e^*+\varepsilon)\, u'(W_\theta(e^*+\varepsilon))g_\theta(\varepsilon)d\varepsilon = C'(e^*).$$

When precision increases in an MPS sense, the distribution of ε can be divided into three regions: the left tail, the right tail, and the centre. Thus, there exist ε_a and ε_b such that

$$\frac{dg_\theta(\varepsilon)}{d\theta} \begin{cases} > 0 & \text{if } \varepsilon \in (\varepsilon_a, \varepsilon_b), \\ \leq 0 & \text{if } \varepsilon \notin (\varepsilon_a, \varepsilon_b), \end{cases} \tag{54}$$

An increase in θ shifts mass away from the left tail ($\varepsilon \leq \varepsilon_a$) and right tail ($\varepsilon \geq \varepsilon_b$) and towards the centre ($\varepsilon_a < \varepsilon < \varepsilon_b$). Similar to equation (9) in the core model, the incentive effect is positive if and only if

$$\frac{\partial^2}{\partial e \partial \theta}\mathbb{E}_\theta\left[u\left(W_\theta(q)\right)|e\right] \geq 0.$$

Proposition 6 gives the effect of precision on incentives when we allow for risk aversion, although it also holds for the case of risk neutrality.

Proposition 6 *(Effect of precision on incentives, risk aversion) (i) When the limited liability constraint is binding for all $q < e^*+\varepsilon_b$, then effort and precision are substitutes.*

(ii) When the limited liability constraint is binding for all $q < e^*+\varepsilon_a$, let $q^* \geq e^*+\varepsilon_a$ be the level of output such that the limited liability constraint is binding if and only if

[24]With a location parameter, output q is equal to effort plus white noise. By construction, g_θ is the PDF of the white noise.

$q < q^*$. Then, effort and precision are complements if and only if q^* is such that $q^* < \widehat{q}$.

The intuition is as follows. In general, the effect of precision on effort is complex, because there are up to three relevant regions. Part (i) studies the case in which limited liability binds for all $q < e^* + \varepsilon_b$, and so the wage is positive only in the right tail. An increase in precision reduces the right tail and thus lowers incentives, regardless of whether the agent is risk-neutral or risk-averse. The intuition is as in the core model. An increase in precision reduces the likelihood that output ends up in the right tail, and thus the agent is rewarded for increases in effort.

Part (ii) studies the case in which limited liability binds for all $q < e^* + \varepsilon_a$, and so the wage is positive only in the centre and right tail. We define q^* as the threshold output above which the wage is positive, analogous to the threshold X_θ in the core model. Effort and precision are complements if and only if this threshold is below a cutoff $\widehat{q}$ – analogous to the condition $X_\theta < \widehat{X}$ in Proposition 2. Again, the intuition is as in the core model. If the threshold q^* is low, then the agent will be paid for any increases in effort unless he suffers a sufficiently negative shock to push output below the threshold. An increase in precision reduces the risk of such shocks, and so raises incentives. If the threshold q^* is high, the agent will be paid for any increases in effort only if he enjoys a sufficiently positive shock to push output above the threshold. An increase in precision reduces the likelihood of such shocks, and so reduces incentives.[25]

C.1 Log utility and linear likelihood ratio

With risk aversion, in general it is not possible to pin down the shape of the optimal contract – it may be convex, concave, or have both convex and concave regions. However, as shown by Chaigneau, Edmans, and Gottlieb (2016), if the agent has log

[25] If limited liability does not bind for all $q < e^* + \varepsilon_a$, i.e. the wage is positive in part of the left tail (as well as the centre and right tails), one might think that precision and effort are always complements, similar to the core model for low X_θ – the wage is positive except for very low output realizations, and increases in precision reduce the risk of such output realizations. However, this need not be the case. When the wage is positive in the left tail, we must consider how precision affects incentives in the left tail (not just the center and right tail). As discussed at the end of Section C.1, a risk-averse agent's incentives are given by utility-adjusted pay-performance sensitivity. Due to diminishing marginal utility, the agent places higher weight on (dollar) pay-performance sensitivity in the left tail than at the center or in the right tail. While an increase in precision redistributes probability mass from the right tail towards the center, it also redistributes mass from the left tail to the center, and so may reduce incentives. Thus, the effect of precision on incentives is ambiguous when limited liability does not bind for all $q < e^* + \varepsilon_a$.

utility and the likelihood ratio is linear (as with the (truncated) normal and gamma distributions), the optimal contract is an option. This result is stated in Proposition 7.

Proposition 7 *(Log utility, linear likelihood ratio) With log utility and a likelihood ratio linear in q, the optimal contract gives the agent $\hat{b}_\theta$ options with strike price q^*:*

$$W_\theta\left(q\right) = \hat{b}_\theta \max\{q - q^*, 0\}. \tag{55}$$

Moreover, if the output distribution is symmetric, these options are at-the-money: $q^* = \mathbb{E}[q|e^*] = e^*$. Then, an increase in precision does not change the effort incentives of a risk neutral agent, but it increases the effort incentives of a risk averse agent.

The optimal contract gives the agent $\hat{b}_\theta$ options on q with strike price q^*, which is such that the likelihood ratio is equal to zero at $q = q^*$: the agent gets a positive wage whenever the likelihood ratio is positive. Expected firm value is always $\mathbb{E}[q|e^*] = e^*$, so that an option with strike price e^* is at-the-money. Moreover, if we assume that the output distribution is symmetric (as with the normal distribution), then the level of output such the likelihood ratio turns positive is $q^* = e^*$. In this case, the optimal contract involves at-the-money options.

With an at-the-money option and a symmetric distribution, an increase in precision does not change the likelihood that the option ends up in-the-money (which remains at 1/2). Thus, it does not change the expected pay-performance sensitivity of the option, and therefore effort incentives – regardless of precision, if the agent increases effort by 1 unit, there is a 1/2 probability that the option ends up in-the-money, in which case the incremental effort increases his pay by $\hat{b}_\theta$ units. With risk aversion, effort incentives depend on the *utility-adjusted* pay-performance sensitivity. An at-the-money option contract gives the agent a constant reward of $\hat{b}_\theta$ for every one unit increase in output above expected output (e^*). Due to diminishing marginal utility, a reward of $\hat{b}_\theta$ has less effect on the agent's utility at high levels of wealth (i.e., in the right tail) than at low levels of wealth (i.e., in the center). An increase in precision redistributes probability mass from the right tail to the centre, i.e., from low to high utility-adjusted pay-performance sensitivity regions, and thus raises effort incentives.

C.2 Proofs

Proof of Proposition 6

We start with part (i). For a given contract, the effect of a marginal increase in precision on the LHS of the incentive constraint is given by:

$$\int_{-\infty}^{\infty} W_{\theta}'(e^* + \varepsilon)\, u'(W_{\theta}(e^* + \varepsilon))\frac{dg_{\theta}(\varepsilon)}{d\theta}d\varepsilon. \tag{56}$$

When limited liability is binding for $q < \varepsilon_b + e^*$, $W_{\theta}(e^* + \varepsilon) = 0$ for $\varepsilon < \varepsilon_b$. Thus, (56) becomes:

$$\int_{\varepsilon_b}^{\infty} W_{\theta}'(e^* + \varepsilon)\, u'(W_{\theta}(e^* + \varepsilon))\frac{dg_{\theta}(\varepsilon)}{d\theta}d\varepsilon. \tag{57}$$

Whether the agent is risk-neutral or risk-averse, we have $u' > 0$, and MLRP ensures $W_{\theta}' > 0$. In addition, from the definition of ε_b in (54), $\frac{dg_{\theta}(\varepsilon)}{d\theta} \leq 0$ for $\varepsilon > \varepsilon_b$, so (57) is negative.

We now turn to part (ii). For a given contract, when limited liability is binding for $q < q^*$, where $q^* \geq e^* + \varepsilon_a$, the effect of a marginal increase in precision on the LHS of the incentive constraint is given by:

$$\int_{q^*-e^*}^{\varepsilon_b} W_{\theta}'(e^* + \varepsilon)\, u'(W_{\theta}(e^* + \varepsilon))\frac{dg_{\theta}(\varepsilon)}{d\theta}d\varepsilon + \int_{\varepsilon_b}^{\infty} W_{\theta}'(e^* + \varepsilon)\, u'(W_{\theta}(e^* + \varepsilon))\frac{dg_{\theta}(\varepsilon)}{d\theta}d\varepsilon. \tag{58}$$

As above, $u' > 0$ and $W_{\theta}' > 0$. Moreover, $\frac{dg_{\theta}(\varepsilon)}{d\theta}$ is positive under the first integral and negative under the second integral. The expression in (58) is therefore strictly decreasing in q^*, and strictly negative for $q^* \geq e^* + \varepsilon_b$. If there exists $q^* \in [e^* + \varepsilon_a, e^* + \varepsilon_b]$ such that the expression in (58) is equal to zero, then let $\widehat{q}$ be equal to this value of q^*. If such a value does not exist, i.e., the expression in (58) is strictly negative for $q^* \geq e^* + \varepsilon_a$, then let $\widehat{q} = -\infty$.

Proof of Proposition 7

A linear likelihood ratio can be written as $\frac{f_e(q|e^*)}{f(q|e^*)} \equiv a + bq$ with $b > 0$ due to MLRP. In addition, log utility yields $u'^{-1}(W) = \frac{1}{W}$. Thus, (53) can be written as

$$W_{\theta}(q) = \begin{cases} \hat{b}_{\theta}(q - q^*) & \text{if } q \geq q^*, \\ 0 & \text{if } q < q^*, \end{cases} \tag{59}$$

where $\hat{b}_{\theta} = \mu b > 0$ and $q^* = -\frac{a}{b}$ (which implies that $\hat{b}_{\theta}(q - q^*) = 0$ at $q = q^*$). Equation

(59) can thus be rewritten as in (55).

With MLRP, there is only one value q^* such that $\frac{f_e(q^*|e^*)}{f(q^*|e^*)} = 0$. For a distribution with a location parameter, this equality implies that the derivative of the PDF with respect to q is equal to zero at q^*, and at this point only. If the distribution is symmetric, and if the derivative of the PDF is equal to zero at a single point, this point must be the point around which distribution is centered, i.e., it must be the mean of the distribution e^*. That is, for a symmetric distribution with a location parameter that satisfies MLRP, the likelihood ratio $\frac{f_e(q|e^*)}{f(q|e^*)}$ is only equal to zero at $q = e^*$. With a linear likelihood ratio, this implies $a + be^* = 0$, so that $q^* = e^*$.

In this case, the expression in (58) can be rewritten as

$$\int_0^\infty \hat{b}_\theta u'(\hat{b}_\theta(e^* + \varepsilon - e^*)) \frac{dg_\theta(\varepsilon)}{d\theta} d\varepsilon. \tag{60}$$

With a risk-neutral agent, u' is a positive constant, and this expression has the same sign as $\int_0^\infty \frac{dg_\theta(\varepsilon)}{d\theta} d\varepsilon$, which is equal to zero because a change in θ is a MPS and the distribution of ε is symmetric around zero for any θ. With a risk-averse agent, the expression in (60) can be rewritten as

$$\int_0^{\varepsilon_b} \hat{b}_\theta u'(\hat{b}_\theta(e^* + \varepsilon - e^*)) \frac{dg_\theta(\varepsilon)}{d\theta} d\varepsilon + \int_{\varepsilon_b}^\infty \hat{b}_\theta u'(\hat{b}_\theta(e^* + \varepsilon - e^*)) \frac{dg_\theta(\varepsilon)}{d\theta} d\varepsilon. \tag{61}$$

As above, $\int_0^\infty \frac{dg_\theta(\varepsilon)}{d\theta} d\varepsilon = 0$, and the first integral in (61) is positive while the second is negative because of (54). With a risk-averse agent, $u'' < 0$ and so the first integral outweighs the second, so that the expression in (61) is positive.

D Location-scale distributions

Claim 1 *For distributions parametrized with e and σ such that $F_\sigma(q|e) = G\left(\frac{q-e}{\sigma}\right)$, the option vega is highest when $X_\sigma = e$.*

Proof. The agent's expected pay under volatility σ and effort e is given by

$$\mathbb{E}[W_\sigma(q)|e] = \int_{X_\sigma}^\infty (q - X_\sigma) f_\sigma(q|e) dq. \tag{62}$$

Using the same integration by parts as in (10) yields

$$\mathbb{E}\left[W_\theta\left(q\right)|e\right] = \mathbb{E}[q|e] - X_\sigma + \int_{-\infty}^{X_\sigma} F_\sigma\left(q|e\right) dq. \tag{63}$$

Thus, the vega of the option is

$$\nu = \frac{\partial}{\partial\sigma}\mathbb{E}\left[W_\theta\left(q\right)|e\right] = \frac{\partial}{\partial\sigma}\left\{\mathbb{E}[q|e] - X_\sigma + \int_{-\infty}^{X_\sigma} F_\sigma\left(q|e\right) dq\right\}.$$

Since $F_\sigma\left(q|e\right) = G\left(\frac{q-e}{\sigma}\right)$, we have

$$\nu = \frac{\partial}{\partial\sigma}\left\{\int_{-\infty}^{X_\sigma} G\left(\frac{q-e}{\sigma}\right) dq\right\} = -\frac{1}{\sigma}\int_{-\infty}^{X_\sigma} \frac{q-e}{\sigma} g\left(\frac{q-e}{\sigma}\right) dq.$$

Using the change of variables $y = \frac{q-e}{\sigma}$ gives

$$\nu = \int_{-\infty}^{\frac{X_\sigma - e}{\sigma}} (-y)g(y)dq. \tag{64}$$

Since $g(y) > 0$, this expression is maximized for $X_\sigma = e$, i.e., for an at-the-money option.[26] ■

Claim 2 shows that, for symmetric distributions, the vegas of the option-when-working and option-when-shirking are equal for $X_\sigma = \frac{\underline{e}+\overline{e}}{2}$.

Claim 2 *For symmetric distributions parametrized by e and σ such that $F_\sigma\left(q|e\right) = G\left(\frac{q-e}{\sigma}\right)$, the vegas of the option-when working and the option-when-shirking are equal for $X_\sigma = \frac{\underline{e}+\overline{e}}{2}$.*

Proof. Using (64), for $X_\sigma = \frac{\underline{e}+\overline{e}}{2}$, the vega $\nu_{\overline{e}}$ of the option-when-working ($e = \overline{e}$) is

$$\nu_{\overline{e}} = \int_{-\infty}^{\frac{X_\sigma - \overline{e}}{\sigma}} (-y)g(y)dq = \int_{-\infty}^{\frac{\underline{e}-\overline{e}}{2\sigma}} (-y)g(y)dq.$$

[26] With high effort, $e = \overline{e}$, so the option-when-working is ATM for $X_\sigma = \overline{e}$. With low effort, $e = \underline{e}$, so the option-when-shirking is ATM for $X_\sigma = \underline{e}$.

Likewise, for $X_\sigma = \frac{\underline{e}+\overline{e}}{2}$, the vega $\nu_{\underline{e}}$ of the option-when-shirking ($e = \underline{e}$) is

$$\nu_{\underline{e}} = \int_{-\infty}^{\frac{X_\sigma - \underline{e}}{\sigma}} (-y)g(y)dq = \int_{-\infty}^{\frac{\overline{e}-\underline{e}}{2\sigma}} (-y)g(y)dq.$$

In addition,

$$\int_{-\infty}^{\frac{\overline{e}-\underline{e}}{2\sigma}} (-y)g(y)dq = \int_{-\infty}^{\frac{\underline{e}-\overline{e}}{2\sigma}} (-y)g(y)dq + \int_{\frac{\underline{e}-\overline{e}}{2\sigma}}^{\frac{\overline{e}-\underline{e}}{2\sigma}} (-y)g(y)dq. \tag{65}$$

For a symmetric distribution, we have $\int_{-\frac{\overline{e}-\underline{e}}{2\sigma}}^{\frac{\overline{e}-\underline{e}}{2\sigma}} (-y)g(y)dq = 0$. Equation (65) then implies that $\nu_{\overline{e}} = \nu_{\underline{e}}$. ■

E Normal distribution

This section provides analytical results to support the graph in Figure 1, which illustrates the direct and incentive effects for the Normal distribution. The proofs are in Section E.1.

Let φ and Φ denote the PDF and CDF of the standard normal distribution, respectively. As shown in Appendix E.1, the total and direct effects are respectively given by:

$$\frac{d\mathbb{E}\left[W_\sigma\left(q\right)|\overline{e}\right]}{d\sigma} = \varphi\left(\frac{X_\sigma - \overline{e}}{\sigma}\right) - \left[1 - \Phi\left(\frac{X_\sigma - \overline{e}}{\sigma}\right)\right] \frac{\varphi\left(\frac{X_\sigma - \overline{e}}{\sigma}\right) - \varphi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right) - \Phi\left(\frac{X_\sigma - \overline{e}}{\sigma}\right)}, \text{ and} \tag{66}$$

$$\frac{\partial\mathbb{E}\left[W_\sigma\left(q\right)|\overline{e}\right]}{\partial\sigma} = \varphi\left(\frac{X_\sigma - \overline{e}}{\sigma}\right). \tag{67}$$

The direct effect is discussed in the main text. The incentive effect, $\frac{\partial\mathbb{E}[W_\sigma(q)|\overline{e}]}{\partial X_\sigma}\frac{dX_\sigma}{d\sigma}$, consists of two components. The first is the change in strike price required to maintain incentive compatibility, $\frac{dX_\sigma}{d\sigma}$. From Corollary 1 and using $\sigma = \frac{1}{\theta}$, $\frac{dX_\sigma}{d\sigma} > 0$ if and only if $X_\sigma > \widehat{X} = \frac{\overline{e}+\underline{e}}{2}$. Indeed, for the normal distribution, not only does $\frac{dX_\sigma}{d\sigma}$ turn from negative to positive as X_σ crosses above $\widehat{X}$, but it is also monotonically increasing in X_σ, i.e., monotonically decreasing in the cost of effort. This result is stated in Lemma 6 below:

Lemma 6 *(Normal distribution, change in strike price): Suppose ε is normally distributed. Then, the effect of volatility on the strike price is decreasing in the cost of effort, i.e.,*

$$\frac{d^2 X_\sigma}{d\sigma dC} < 0. \tag{68}$$

The second component is the change in the value of the option when the strike price increases, $\frac{\partial \mathbb{E}[W_\sigma(q)|\overline{e}]}{\partial X_\sigma}$. This change is always negative, and its negativity is increasing in the moneyness of the option. Overall, as X falls below $\widehat{X}$ and the option becomes increasingly in the money, both $\frac{dX_\sigma}{d\sigma}$ and $\frac{\partial \mathbb{E}[W_\sigma(q)|\overline{e}]}{\partial X_\sigma}$ become increasingly negative, and so the overall incentive effect $\frac{\partial \mathbb{E}[W_\sigma(q)|\overline{e}]}{\partial X_\sigma}\frac{dX_\sigma}{d\sigma}$ becomes monotonically more positive. However, as X rises above $\widehat{X}$, the two components of the incentive effect move in opposite directions. On the one hand, greater precision increasingly worsens incentives ($\frac{dX_\sigma}{d\sigma}$ becomes more positive). On the other hand, $\frac{\partial \mathbb{E}[W_\sigma(q)|\overline{e}]}{\partial X_\sigma}$ rises towards zero: when the option is deeply out-of-the-money, its value is small to begin with and thus little affected by the strike price. Overall, the impact of X on incentives is non-monotonic. As X initially rises above $\widehat{X}$, the incentive effect becomes increasingly negative but subsequently rises to zero.

Proposition 8 proves for the normal distribution that the value of information is monotonically increasing in C (the exogenous parameter that drives X).

Proposition 8 *(Normal distribution, effect of cost of effort on value of information) Suppose ε is normally distributed. Then,* $\frac{d}{dC}\left\{\frac{d\mathbb{E}[W_\sigma(q)|\overline{e}]}{d\sigma}\right\} > 0$.

E.1 Proofs

Proof of Equations (66) and (67)

First, with σ instead of θ, the decomposition in (8) can be rewritten as

$$\frac{d}{d\sigma}\mathbb{E}\left[W_\sigma(q)\,|\overline{e}\right] = \underbrace{\frac{\partial}{\partial\sigma}\mathbb{E}\left[W_\sigma(q)\,|\overline{e}\right]}_{\textit{direct effect}} + \underbrace{\frac{\partial}{\partial X_\sigma}\mathbb{E}\left[W_\sigma(q)\,|\overline{e}\right]\frac{dX_\sigma}{d\sigma}}_{\textit{incentive effect}}. \tag{69}$$

Second,

$$\begin{aligned}
\frac{\partial \mathbb{E}[W_\sigma(q)|e]}{\partial \sigma} &= \frac{\partial}{\partial \sigma}\int_{X_\sigma}^{\infty}(q-X_\sigma)\frac{1}{\sigma}\varphi\left(\frac{q-e}{\sigma}\right)dq = \frac{\partial}{\partial \sigma}\int_{X_\sigma-e}^{\infty}\frac{q+e-X_\sigma}{\sigma}\varphi\left(\frac{q}{\sigma}\right)dq \\
&= \frac{\partial}{\partial \sigma}\int_{X_\sigma-e}^{\infty}\frac{q}{\sigma}\varphi\left(\frac{q}{\sigma}\right)dq - (X_\sigma - e)\frac{\partial}{\partial \sigma}\int_{X_\sigma-e}^{\infty}\frac{1}{\sigma}\varphi\left(\frac{q}{\sigma}\right)dq \\
&= \frac{\partial}{\partial \sigma}\left\{\left[-\sigma\varphi\left(\frac{q}{\sigma}\right)\right]_{X_\sigma-e}^{\infty}\right\} - (X_\sigma - e)\frac{\partial}{\partial \sigma}\left\{1-\Phi\left(\frac{X_\sigma-e}{\sigma}\right)\right\} \\
&= \varphi\left(\frac{X_\sigma-e}{\sigma}\right) - \sigma\frac{X_\sigma-e}{\sigma^2}\varphi'\left(\frac{X_\sigma-e}{\sigma}\right) + (X_\sigma-e)\left(-\frac{X_\sigma-e}{\sigma^2}\right)\varphi\left(\frac{X_\sigma-e}{\sigma}\right) \\
&= \varphi\left(\frac{X_\sigma-e}{\sigma}\right) - \frac{X_\sigma-e}{\sigma}\varphi'\left(\frac{X_\sigma-e}{\sigma}\right) + \frac{X_\sigma-e}{\sigma}\varphi'\left(\frac{X_\sigma-e}{\sigma}\right) = \varphi\left(\frac{X_\sigma-e}{\sigma}\right).
\end{aligned}$$

where the fourth and sixth equalities use $\varphi'(x) = -x\varphi(x)$, and the fifth equality uses $\varphi(x) \to_{x\to\infty} 0$. This establishes (67). In addition, it follows that

$$\frac{\partial}{\partial \sigma}\left\{\mathbb{E}\left[W_\sigma(q)|\bar{e}\right] - \mathbb{E}\left[W_\sigma(q)|\underline{e}\right]\right\} = \varphi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right) - \varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right). \tag{70}$$

Third,

$$\begin{aligned}
\frac{\partial \mathbb{E}[W_\sigma(q)|e]}{\partial X_\sigma} &= \frac{\partial}{\partial X_\sigma}\int_{X_\sigma}^{\infty}(q-X_\sigma)\frac{1}{\sigma}\varphi\left(\frac{q-e}{\sigma}\right)dq \\
&= \int_{X_\sigma}^{\infty}-\frac{1}{\sigma}\varphi\left(\frac{q-e}{\sigma}\right)dq = -\left(1-\Phi\left(\frac{X_\sigma-e}{\sigma}\right)\right). \qquad (71)
\end{aligned}$$

It follows that

$$\begin{aligned}
\frac{\partial}{\partial X_\sigma}\left\{\mathbb{E}\left[W_\sigma(q)|\bar{e}\right] - \mathbb{E}\left[W_\sigma(q)|\underline{e}\right]\right\} &= -\left(1-\Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)\right) + \left(1-\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\right) \\
&= \Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right) - \Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right). \qquad (72)
\end{aligned}$$

which is strictly negative because of MLRP, which implies FOSD.

Fourth, according to Lemma 1, following a change in σ the strike price X_σ adjusts so that the incentive constraint remains satisfied as an equality, so:

$$\frac{\partial\left\{\mathbb{E}[W_\sigma(q)|\bar{e}] - \mathbb{E}[W_\sigma(q)|\underline{e}]\right\}}{\partial \sigma} + \frac{\partial\left\{\mathbb{E}[W_\sigma(q)|\bar{e}] - \mathbb{E}[W_\sigma(q)|\underline{e}]\right\}}{\partial X_\sigma}\frac{dX_\sigma}{d\sigma} = 0.$$

Rearranging and using the results in equations (70) and (72):

$$\frac{dX_\sigma}{d\sigma} = -\frac{\varphi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}. \tag{73}$$

Using the results above, we can rewrite (69) as

$$\frac{d\mathbb{E}[W_\sigma(q)\,|\bar{e}]}{d\sigma} = \varphi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)+\left[1-\Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)\right]\frac{\varphi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}. \tag{74}$$

This establishes (66).

Proof of Lemma 6

As X_σ is strictly decreasing in C (see Proposition 4), inequality (68) holds if and only if $\frac{d\frac{dX_\sigma}{d\sigma}}{dX} > 0$. As established in the proof of equations (66) and (67) above,

$$\frac{dX_\sigma}{d\sigma} = -\frac{\varphi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}.$$

To simplify notation, define

$$x \equiv \frac{X_\sigma-\underline{e}}{\sigma}, t \equiv \frac{\bar{e}-\underline{e}}{\sigma}.$$

We wish to show that $\forall t > 0$,

$$f(x,t) \equiv [\varphi(x)-\varphi(x-t)]^2 - [\Phi(x)-\Phi(x-t)][\varphi'(x)-\varphi'(x-t)] > 0, \quad \forall x, \tag{75}$$

where

$$\varphi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$$

$$\Phi(x) = \int_{-\infty}^{x}\varphi(y)\,dy.$$

For $t = 0$, $f(x,0)$ is trivially 0. Since $\varphi(x) = \varphi(-x)$, we have $\Phi(x) - \Phi(x-t) = \Phi(-x+t) - \Phi(-x)$ and $\varphi'(x) - \varphi'(x-t) = \varphi'(-x+t) - \varphi'(-x)$. As a consequence, $f(x,t) = f(-x+t,t)$. We thus only have to study $x \geq \frac{t}{2} > 0$.

We first analyze the term $\varphi'(x) - \varphi'(x-t)$. Since

$$\varphi'(x) = -\frac{x}{\sqrt{2\pi}} e^{-\frac{x^2}{2}},$$

$$\varphi'(x) - \varphi'(x-t) = \varphi(x-t)(-xe^{-t(x-t/2)} + x - t).$$

When $x \geq t/2$, the function $e^{-t(x-t/2)} - 1 + \frac{t}{x}$ is only equal to zero at one point, since it monotonically decreases from 2 to -1. Let that point be x_0. Then

$$\varphi'(x) - \varphi'(x-t) \begin{cases} < 0 & \frac{t}{2} \leq x < x_0 \\ = 0 & x = x_0 \\ > 0 & x > x_0 \end{cases}.$$

We know that when $x \in [\frac{t}{2}, x_0]$, $f(x,t) > 0$ since $[\varphi(x) - \varphi(x-t)]^2 > 0$ and $\Phi(x) - \Phi(x-t) > 0\ \forall x$, so that (75) is proven for $x \in [\frac{t}{2}, x_0]$

We now prove (75) for $x > x_0$. In this interval (omitting the argument t):

$$f(x,t) > 0 \iff g(x) \equiv \frac{f(x,t)}{\varphi'(x) - \varphi'(x-t)} > 0.$$

To prove the latter, we first calculate

$$\begin{aligned} g'(x) &= \frac{2[\varphi(x) - \varphi(x-t)][\varphi'(x) - \varphi'(x-t)]^2 - [\varphi(x) - \varphi(x-t)]^2[\varphi''(x) - \varphi''(x-t)]}{[\varphi'(x) - \varphi'(x-t)]^2} \\ &\quad -[\varphi(x) - \varphi(x-t)] \\ &= \frac{[\varphi(x) - \varphi(x-t)]\varphi(x-t)^2}{[\varphi'(x) - \varphi'(x-t)]^2} \left[\left(e^{-t(x-t/2)} - 1 \right)^2 + t^2 e^{-t(x-t/2)} \right] \\ &< 0, \quad x \in (x_0, \infty), \end{aligned}$$

where in the last step we used the fact that $\varphi(x) < \varphi(x-t)$ when $x > t/2$. Therefore,

$$g(x) > 0 \quad \forall x \in (x_0, \infty) \iff \lim_{x \to \infty} g(x) \geq 0.$$

Since

$$g(x) = \frac{[\varphi(x) - \varphi(x-t)]^2}{\varphi'(x) - \varphi'(x-t)} - \Phi(x) + \Phi(x-t)$$
$$= \frac{1}{\sqrt{2\pi}} e^{-(x-t)^2/2} \frac{\left(e^{-t(x-t/2)} - 1\right)^2}{-xe^{-t(x-t/2)} + x - t} - \Phi(x) + \Phi(x-t),$$

it is clear that

$$\lim_{x\to\infty} g(x) = 0.$$

Proof of Proposition 8

Using the chain rule,

$$\frac{d}{dC}\left\{\frac{d\mathbb{E}[W_\sigma(q)\,|\bar{e}]}{d\sigma}\right\} = \frac{d}{dX_\sigma}\left\{\frac{d\mathbb{E}[W_\sigma(q)\,|\bar{e}]}{d\sigma}\right\}\frac{dX_\sigma}{dC}$$

Since $\frac{dX_\sigma}{dC} < 0$ (Lemma 2), we have $\frac{d}{dC}\left\{\frac{d\mathbb{E}[W_\sigma(q)|\bar{e}]}{d\sigma}\right\} > 0$ if and only if $\frac{d}{dX_\sigma}\left\{\frac{d\mathbb{E}[W_\sigma(q)|\bar{e}]}{d\sigma}\right\} <$ 0.

Using (66) and $\varphi'(x) = -x\varphi(x)$ for the normal distribution, we have

$$\frac{d}{dX_\sigma}\left\{\frac{d\mathbb{E}[W_\sigma(q)\,|\bar{e}]}{d\sigma}\right\} = \frac{d}{dX_\sigma}\left\{\varphi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right) - \left[1-\Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)\right]\frac{\varphi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)}\right\}$$

$$\begin{aligned}
= \frac{1}{\sigma}\Bigg(& -\frac{X_\sigma-\bar{e}}{\sigma}\varphi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right) + \left[\frac{X_\sigma-\bar{e}}{\sigma}\varphi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right) - \frac{X_\sigma-\underline{e}}{\sigma}\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\right]\frac{1-\Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)} \\
& + \left[\varphi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right) - \varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\right]\frac{\varphi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)} \\
& - \frac{1-\Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)}{\left(\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)\right)^2}\left(\varphi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\right)^2\Bigg) \qquad (76)
\end{aligned}$$

Multiplying all terms by $\sigma\left(\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\bar{e}}{\sigma}\right)\right) > 0$, the expression in (76) has the same sign as

$$\left[\frac{\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)}-\frac{X_\sigma-\underline{e}}{\sigma}\right]\left[\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\left[1-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)\right]\right.$$
$$\left.-\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)\left[1-\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\right]\right]-\frac{\overline{e}-\underline{e}}{\sigma}\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)\left[1-\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\right]. \quad (77)$$

Since the last term in (77) is always negative, the expression in (77) is negative if the first line in (77) is negative. We now prove the latter.

The hazard rate $\varphi(x)/(1-\Phi(x))$ of the normal distribution is increasing, which implies that

$$\frac{\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{1-\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}>\frac{\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)}{1-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)}.$$

Rearranging, we have

$$\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\left[1-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)\right]-\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)\left[1-\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\right]>0 \quad (78)$$

Define

$$d(X_\sigma,\overline{e})\equiv\frac{\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)}.$$

If $d(X_\sigma,\overline{e})<\frac{X_\sigma-\underline{e}}{\sigma}$, then combining with (78) establishes that (77) is negative, as desired. We now show that $d(X_\sigma,\overline{e})<\frac{X_\sigma-\underline{e}}{\sigma}$, by proving first that $d(X_\sigma,\overline{e})\longrightarrow_{\overline{e}\to 0}\frac{X_\sigma-\underline{e}}{\sigma}$ and second that $d(X_\sigma,\overline{e})$ is decreasing in $\overline{e}$.

First,

$$\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)=-\varphi'\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\frac{\overline{e}-\underline{e}}{\sigma}+O(\overline{e}^2)$$
$$\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)=\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\frac{\overline{e}-\underline{e}}{\sigma}+O(\overline{e}^2).$$

Using $\varphi'(x)=-x\varphi(x)$ for the normal distribution, we have

$$\frac{\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)}\longrightarrow_{\overline{e}\to\underline{e}}\frac{\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\frac{(\overline{e}-\underline{e})(X_\sigma-\underline{e})}{\sigma^2}}{\frac{\overline{e}-\underline{e}}{\sigma}\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}=\frac{X_\sigma-\underline{e}}{\sigma}.$$

Second,

$$\frac{d}{d\overline{e}}\left\{\frac{\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)}\right\}=\frac{d}{d\overline{e}}\left\{\frac{\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma} q\exp\left\{-\frac{q^2}{2}\right\}dq}{\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma}\exp\left\{-\frac{q^2}{2}\right\}dq}\right\}$$

$$=\frac{1}{\sigma}\frac{\frac{X_\sigma-\overline{e}}{\sigma}\exp\left\{-\frac{(X_\sigma-\overline{e})^2}{2\sigma^2}\right\}\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma}\exp\left\{-\frac{q^2}{2}\right\}dq-\exp\left\{-\frac{(X_\sigma-\overline{e})^2}{2\sigma^2}\right\}\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma} q\exp\left\{-\frac{q^2}{2}\right\}dq}{\left(\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma}\exp\left\{-\frac{q^2}{2}\right\}dq\right)^2}.$$

This expression has the same sign as

$$\frac{X_\sigma-\overline{e}}{\sigma}\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma}\exp\left\{-\frac{q^2}{2}\right\}dq-\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma} q\exp\left\{-\frac{q^2}{2}\right\}dq$$

$$=\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma}\left[\frac{X_\sigma-\overline{e}}{\sigma}-q\right]\exp\left\{-\frac{q^2}{2}\right\}dq<0.$$

This establishes that $d(X_\sigma,\overline{e})$ is decreasing in $\overline{e}$, which completes the proof.

F Principal's participation constraint binds

In the core model, the principal has full bargaining power and offers the contract to the agent. The opposite assumption is for the agent to have full bargaining power and offer the contract to the principal, as in Innes (1990). Under this assumption, the agent (entrepreneur) raises an amount $I>0$ from the principal (investor) to fund a project which produces output q. Now, it is the principal's, rather than the agent's, participation constraint that binds. We also assume that it is the agent who now chooses precision, since it is he that captures the surplus and thus bears any gains or losses from the incentive effect.

As discussed in Section 4.1, the optimal contract involves the principal receiving risky debt with face value X_θ. The entrepreneur, who holds equity in a levered firm, chooses X_θ to maximize his payoff, subject to his incentive constraint (79), the investor's participation constraint (80), and the contracting constraints (2) and (3):

$$\max_{X_\theta} \int_{X_\theta}^{\infty} (q - X_\theta) f_\theta(q|e^*)dq - C(e^*)$$

$$\text{subject to} \quad e^* \in \arg\max_e \int_{-\infty}^{\infty} W_\theta(q) f_\theta(q|e)dq - C(e), \tag{79}$$

$$\int_{-\infty}^{X_\theta} q f_\theta(q|e^*)dq + \int_{X_\theta}^{\infty} X_\theta f_\theta(q|e^*)dq = I. \tag{80}$$

We assume that there exists a face value X_θ and associated effort e^* such that the LHS of (80) is at least I (i.e., an optimal contract exists). Since $I > 0$, we have $X_\theta > 0$.

For a given debt contract, higher precision increases the investor's expected payoff on the LHS of (80) – lower risk increases the value of risky debt. This is the analog of the "direct effect" in the core model. This in turn allows the entrepreneur to reduce X_θ while still satisfying the investor's participation constraint, fully offsetting the direct effect. In turn, a lower X_θ increases the entrepreneur's effort incentives. This is a similar participation effect to Section 4.2 and means that the value of information again stems entirely from its effect on effort, which remains (26).

There are two differences when it is the agent rather than the principal who offers the contract and chooses precision. The first is that the level of precision chosen will be different. From (25), the value of information to the principal is given by

$$\frac{\partial}{\partial e}\mathbb{E}_\theta\left[R_\theta(q)\,|e_\theta(X_\theta)\right]\frac{de}{d\theta}, \tag{81}$$

where $\frac{de}{d\theta}$, in turn, is given by (26). The value of information to the agent when he offers the contract and chooses precision is given by

$$\frac{\partial}{\partial e}\mathbb{E}_\theta\left[W_\theta(q)\,|e_\theta(X_\theta)\right]\frac{de}{d\theta}. \tag{82}$$

The quantities given by (81) and (82) will typically be different, for the reasons given in the main text. The second difference is that the distinction between ex ante and ex post incentives is not the same as in the core model. If the effort effect is positive, the agent would like to commit to the highest possible level of precision ex ante, as this would maximize the value of the investor's debt claim, allowing him to minimize the face value of debt X_θ. This in turn maximizes his effort incentives, increasing total surplus and his profits (since he has full bargaining power, he captures the full surplus). However, if commitment is not possible and precision is chosen once the contract has been written and effort has been exerted, the agent will select the lowest possible level of θ to maximize the value of his levered equity. As a result, the agent may wish to commit to a high level of precision, for example by accepting covenants in the debt contract. The agent's benefits from precision are always higher ex ante than ex post, whereas the principal's benefits are higher ex ante if and only if the incentive effect is positive, and lower otherwise.[27]

G Social welfare

The core model studies the effect of precision on the principal's profits and, in particular, the agent's effort incentives. This section analyzes the effect of precision on social welfare and, in particular, whether the principal overinvests or underinvests in information relative to the social optimum.

We start with the fixed effort model of Section 3.3.2 as the results are clearest. Total surplus is determined entirely by expected output, $\mathbb{E}_\theta [q|e]$ minus the cost of effort; it does not depend on the wage as this is a mere transfer from the principal to the agent. In turn, expected output depends only on effort and not precision (since changes in precision represent a MPS, $\mathbb{E}_\theta [q|e]$ is independent of θ.) Thus, when effort is fixed, it is independent of precision, and so total surplus is independent of precision. In contrast, the principal has strict incentives to increase precision, because doing so reduces the expected wage: the total effect of precision is positive.[28] As a result, the principal always overinvests in precision: increases in precision have no social benefit but the principal has incentives to undertake them.

We now move to the endogenous effort model. Now, precision may change total surplus by affecting the implemented effort level. Let the principal choose precision at

[27]Note that the analysis of ex ante and ex post incentives when the principal chooses precision was with the fixed effort model. With endogenous effort, the analysis becomes substantially more complex and a pure strategy equilibrium may not even exist. This is because the agent will anticipate the future level of precision, and thus may change his effort choice.

[28]Indeed, if increasing precision were costless, the principal would choose infinite precision. Then, effort of $\overline{e}$ would lead to output of $\overline{e}$ with certainty, and the optimal contract would pay the agent C if $q = \overline{e}$ and zero otherwise. Thus, the agent is paid his cost of effort and does not receive any rents.

a cost $\kappa(\theta)$, where $\kappa(\cdot)$ is an increasing function. Her profit is given by

$$\Pi(\theta) \equiv \max_{X} \left\{ X - \int_{-\infty}^{X} F_\theta(q|e_\theta(X))dq - \kappa(\theta) \right\}.$$

The agent's utility is given by

$$\mathcal{A}(\theta) \equiv \max_{e} \left\{ \mathbb{E}[q|e] - X_\theta + \int_{-\infty}^{X_\theta} F_\theta(q|e)dq - C(e) \right\}$$

and chooses effort to maximize $\mathcal{A}(\theta)$. Total surplus is the sum of profit and utility, i.e.,

$$\mathcal{U}(\theta) \equiv \Pi(\theta) + \mathcal{A}(\theta).$$

Let θ^S (θ^P) be the level of precision that maximizes social welfare (profit). We wish to study whether $\theta^S \gtreqless \theta^P$, i.e., whether the principal underinvests or overinvests in precision relative to the social optimum. Define Z as the weighted average between social welfare and profit:

$$Z(\theta; \alpha) \equiv \alpha \mathcal{U}(\theta) + (1-\alpha)\Pi(\theta),$$

so that Z equals social welfare when $\alpha = 1$ and profit when $\alpha = 0$. The principal overinvests (underinvests) in precision if and only if the level of precision that maximizes Z is decreasing (increasing) in α, i.e., $\theta^S \leq (\geq) \theta^P$ if

$$\frac{\partial^2 Z}{\partial\theta\partial\alpha} \leq (\geq) 0.$$

We have

$$\frac{\partial^2 Z}{\partial\alpha\partial\theta} = \frac{d}{d\theta}\left[\mathcal{U}(\theta) - \Pi(\theta)\right] = \mathcal{A}'(\theta).$$

Thus, the principal overinvests (underinvests) in precision if and only if the agent's utility decreases (increases) with precision, i.e., $A'(\theta) \leq (\geq) 0$.

The intuition is that the principal has two incentives to increase precision. The first is that precision may increase effort. This incentive is fully aligned with the social welfare function. The second is that precision may reduce the agent's wage, redistributing surplus from the agent to herself. This effect has no impact on total surplus. Thus, the principal overinvests in precision if and only if precision reduces the agent's utility.

To study the impact on the agent's utility, note that, by the envelope theorem,

$$\begin{aligned}\mathcal{A}'(\theta) &= \frac{d}{d\theta}\left\{\mathbb{E}[q|e] - X_\theta + \int_{-\infty}^{X_\theta} F_\theta(q|e)dq - C(e)\right\}\Bigg|_{e=e_\theta(X)} \\ &= -\left[1 - F_\theta(X_\theta|e_\theta(X))\right]\frac{dX_\theta}{d\theta} + \int_{-\infty}^{X_\theta} \frac{\partial F_\theta}{\partial\theta}(q|e_\theta(X))dq\end{aligned}$$

Since precision orders F in terms of SOSD, the second term is negative. Holding constant the strike price, a decrease in precision reduces the value of an option – the well-known effect of volatility on option value. If the strike price does not fall with precision ($\frac{dX_\theta}{d\theta} \geq 0$), the first term is non-positive and so the agent's utility falls with precision overall. The agent's utility can only rise with precision if the strike price falls sufficiently to outweigh the impact of reduced volatility.

We apply this analysis to the model from Example 2. Recall from (20) that

$$\begin{aligned} & X^*_{\theta,\kappa} = \frac{1}{\theta} - \frac{1}{\kappa} + \frac{\theta}{2\kappa^2} \\ \implies \quad & \frac{dX^*_{\theta,\kappa}}{d\theta} = -\frac{1}{\theta^2} + \frac{1}{2\kappa^2},\end{aligned}$$

where we needed $\theta \leq 2\kappa$ for the FOA to be valid. Therefore,

$$\frac{dX^*_{\theta,\kappa}}{d\theta} \geq 0 \iff \theta \geq \sqrt{2}\kappa.$$

Thus, when $\theta \leq \sqrt{2}\kappa$, the principal underinvests in precision and when $\sqrt{2}\kappa \leq \theta \leq 2\kappa$, the principal overinvests in precision.

Overall, the force common to both the fixed and endogenous effort models is that the principal overinvests in precision if and only if the agent's utility falls with precision, since this fall is a transfer to the principal with no social benefit. With fixed effort, the agent's utility unambiguously falls with precision and so the principal unambiguously overinvests – intuitively, if agent utility rose with precision, the principal would have added randomness to the initial contract. With endogenous effort, agent utility can rise if the change in precision makes it optimal for the principal to significantly reduce the strike price to implement a higher effort level.

Finally, note that, if the agent's participation constraint is binding, as in Section 4.2, the principal's incentives to invest in precision are fully aligned with social welfare.

This is because, with a binding participation constraint, the agent's rents are zero regardless of the level of precision. Thus, the principal cannot use precision to reduce the agent's rents. Her only motive to increase precision is to increase incentives, which is fully aligned with social welfare.

References

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