Online Appendix for "Financing Through Asset Sales"

B Risky Debt

This section allows the firm to sell debt, in addition to equity and assets. We show first the robustness of the balance sheet effect to allowing for debt issuance, and then the robustness of the camouflage and correlation effects.

We label debt issuance $X = D$. Under this action, the firm offers to repay debtholders a face value P; if it defaults, debtholders confiscate the firm's entire balance sheet. As the model currently stands, the financing decision becomes trivial since the firm can offer risk-free debt with a face value of F, since $F \leq \min(A_L, A_H)$. Thus, to make the problem nontrivial, we assume that risk-free debt capacity has been used up (as in MM) and also introduce a risk of the firm being insolvent. As in Cooley, Marimon, and Quadrini (2004), Carlstrom and Fuerst (1997), Gomes, Yaron, and Zhang (2003), and Hennessy, Livdan, and Miranda (2010), the firm faces the risk of a catastrophic event. Specifically, with probability ("w.p.") p, this event shrinks the firm's balance sheet to a fraction $\alpha \in (0, 1)$ of its original value, where α and p are type-independent and $\alpha E_H < F$, so that debt is risky for all firm types. The shock applies to the firm's entire balance sheet, i.e. both assets in place and the new funds raised, else the firm could again borrow risk-free debt of F. For example, the funds are invested in a zero-NPV project that is subject to the same risks as the firm's technology.[22] Alternatively, as argued by Hennessy, Livdan, and Miranda (2010), the catastrophic event may result from "mass tort claims for defective products or expropriation by a government,"which would apply to the new funds raised even if they were held as cash.

When the non-core asset is sold and separated from the firm, we assume that it continues to face the risk of a shock. For example, the shock could represent market demand or product liability connected with the output produced by the asset. This assumption is non-critical; if the sold assets were not subject to the shock, this would create an additional advantage to asset sales, but the effects that we demonstrate would still apply at the margin.

B.1 Comparison of Debt and Equity

We first examine the choice between debt and equity issuance under positive correlation. In the core model, the firm's fundamental value under equity issuance is $(C_q + A_q + F)(1 - x)$, where x is the fraction of the firm's balance sheet sold to new shareholders. With shocks,

[22] In a model with zero investment return, this assumption implies that financing is negative-NPV, because the firm sells claims with a market value of F but obtains capital worth only $(1 - p + p\alpha)F$ in expectation. As in Section 3, if firms have the option of inaction, we must introduce a sufficiently high investment return to induce them to raise financing; other than that, the following intuition is unchanged. For simplicity, we focus here on the case in which the firm is forced to raise capital.

it becomes $(1-p+p\alpha)(C_q+A_q+F)(1-x)$. New shareholders demand x such that $\mathbb{E}[x(1-p+p\alpha)(C_q+A_q+F)|X=E]=F$, which yields $x=\frac{F}{(1-p+p\alpha)\mathbb{E}[C_q+A_q+F|X=E]}$. Thus fundamental value becomes $(1-p+p\alpha)(C_q+A_q+F)-\left(\frac{C_q+A_q+F}{\mathbb{E}[C+A+F|X=E]}\right)\times F$.

Under debt issuance, fundamental value is $(1-p)(C_q+A_q+F-P)$: w.p. $1-p$, the firm is solvent and pays P; w.p. p, creditors liquidate the firm. Comparing fundamental values, the firm (weakly) prefers debt to equity if and only if

$$\frac{(1-p)P+p\alpha(C_q+A_q+F)}{F}\leq\frac{C_q+A_q+F}{\mathbb{E}[C_q+A_q+F|X=E]} \tag{13}$$

Creditors value their promised payment of P at $(1-p)P+p\alpha\times\mathbb{E}[C+A+F|X=D]$. This intuitively resembles a portfolio of risk-free debt and equity, which is why the core model's results for equity also apply to debt, as we show below. While P is an endogenous variable, it is independent of the firm's type, since it is set by uninformed investors. In equilibrium, creditors demand P such that $F=(1-p)P+p\alpha\times\mathbb{E}[C+A+F|X=D]$. Substituting this expression into the denominator of the LHS, (13) becomes

$$\frac{(1-p)P+p\alpha(C_q+A_q+F)}{(1-p)P+p\alpha\mathbb{E}[C_q+A_q+F|X=D]}\leq\frac{C_q+A_q+F}{\mathbb{E}[C_q+A_q+F|X=E]} \tag{14}$$

Defining $D_q\equiv(1-p)\,P+p\alpha\,(C_q+A_q+F)$, (14) becomes $\frac{D_q}{\mathbb{E}[D|X=D]}\leq\frac{E_q}{\mathbb{E}[E|X=E]}$, i.e. the unit cost of financing is lower for debt than equity, as is intuitive. This inequality will always hold, as in the standard pecking order. Ignoring the conditioning on $X=D$ and $X=E$ and the $(1-p)\,P$ terms on the LHS, both sides of (14) are equal. Adding the $(1-p)\,P$ term to the numerator and denominator of the LHS reduces the unit cost of debt below that of equity and towards 1. Intuitively, adding a probability $1-p$ of a type-independent payment P reduces the information asymmetry of debt relative to that of equity. As a result, given any valuations for debt and equity, H always has the strongest incentive to deviate from equity to debt.[23] Thus, an equity-pooling equilibrium is unsustainable: D1 would require that a debt issuer is inferred as H, and under this inference, H would deviate. Since no semi-separating equilibrium is possible either, only a debt-pooling equilibrium is sustainable.

Inequality (14) provides further intuition about the nature of debt in this model. If $p=1$, then debt is riskless; if $p=0$ (default is certain) then debt is identical to equity. Interestingly, the unit cost of debt financing *falls* as α decreases, because it lowers the information asymmetry over liquidation value. In the limit where $\alpha=0$, liquidation value is zero and the debt claim simply pays P with probability $1-p$, and so there is no information

[23]To see this, rearrange the above inequality to isolate the terms depending on q: A firm prefers debt issuance if and only if $\frac{(1-p)V+p\alpha(C_q+A_q+F)}{C_q+A_q+F}<\frac{(1-p)V+p\alpha\mathbb{E}[C_q+A_q+F|X=D]}{\mathbb{E}[C_q+A_q+F|X=E]}$. The LHS simplifies to $p\alpha+\frac{(1-p)V}{C_q+A_q+F}$ and is thus always minimized for type H.

asymmetry. Finally, the derivative of the unit cost of debt with respect to F is $\left(\frac{p\alpha}{F}\right)^2 (\mathbb{E}[C + A|X = E] - (C_q + A_q))$, which is the same as for equity up to a positive multiple. This expression is positive for $q = L$ and negative for $q = H$, so (as with the equity claim) increasing F lowers the unit cost for type H, because it lowers the amount of information asymmetry in the claim. This is the balance sheet effect, which applies to risky debt as well as equity. As a result, the remaining analysis of risky debt looks very similar to the core model's analysis of equity.

B.2 Comparison of Debt and Asset Sales

Next, we analyze the choice between asset sales and debt issuance, which is similar to the choice between assets and equity. The firm's objective function under asset sales is $(1-p+p\alpha)(C_q + A_q - xA_q(1+k) + F)$, where x is the fraction of assets that must be sold to meet the financing need. Investors set x such that $E[x(1-p+p\alpha)A_q|X = A] = F$, yielding $x = \frac{F}{(1-p+p\alpha)E[A_q|X=A]}$, so the objective function simplifies to $(1-p+p\alpha)(C_q + A_q + F) - F\left(\frac{A_q(1+k)}{E[A_q|X=A]}\right)$. The firm thus prefers debt issuance over asset sales if and only if

$$\frac{(1-p)P + p\alpha(C_q + A_q + F)}{(1-p)P + p\alpha E[C_q + A_q + F|X = D]} \leq \frac{A_q(1+k)}{E[A_q|X = A]}$$

or equivalently

$$\frac{D_q}{E[D|X = D]} \leq \frac{A_q(1+k)}{E[A|X = A]}$$

In the core model, H had a stronger incentive to deviate from an asset-pooling equilibrium to equity issuance if and only if $F \geq F^*$. Here, H has a stronger incentive to deviate from an asset-pooling equilibrium to debt issuance if and only if $\frac{D_H}{D_L} \leq \frac{A_H}{A_L}$, or more explicitly

$$\frac{(1-p)\,P + p\alpha\,(C_H + A_H + F)}{(1-p)\,P + p\alpha\,(C_L + A_L + F)} \leq \frac{A_H}{A_L}. \tag{15}$$

Inequality (15) is the analog of (2) in the core model, and highlights how the intuition for the balance sheet effect is the same. The amount of funds raised, F, only affects the information asymmetry of debt (the LHS) and not of non-core assets (the RHS) as only debt, and not assets, are a claim to the firm's entire balance sheet. Thus, if F is sufficiently high, the LHS falls below 1 and the inequality holds: the information asymmetry of debt becomes sufficiently low that H has the strongest incentive to deviate from an asset-pooling equilibrium.[24] Indeed, (15) yields:

$$F \geq F^{*D} \equiv p\alpha \left(F^{*} + \mathbb{E}[C + A|X = D]\right)(< F^{*}) .$$

This inequality must not hold for the off-equilibrium belief, that a deviator is of type L, to be satisfied. Formally, an asset-pooling equilibrium is sustainable if $F \leq F^{*D}$ (to satisfy D1) and $1 + \overline{k} < \frac{\mathbb{E}[A]}{A_L}$ (as in the core model), and a debt-pooling equilibrium is sustainable if $F \geq F^{*D}$ (to satisfy D1) and $1 + \underline{k} > \frac{D_L}{\mathbb{E}[D]} = 1 - \frac{p\alpha}{F}\left(\mathbb{E}[C + A] - (C_L + A_L)\right)$.[25]

Note that the above result, that debt is preferred for high financing needs and assets are preferred for low financing needs, assumes that a firm has used up its risk-free capacity and that $\alpha E_H < F$, to rule out the trivial case of risk-free debt. If we relax these assumptions and allow for risk-free debt then, as is well-known, this is the preferred claim. Then, debt is preferred for very low financing needs (that are sufficiently low for debt to be risk-free), assets for moderate financing needs, and debt again for high financing needs (due to the balance sheet effect).

A semi-separating equilibrium can be sustained with cutoffs defined analogously to the core model,

$$\frac{A_q(1 + k_q^{*})}{\mathbb{E}[A|X = A]} = \frac{D_q}{\mathbb{E}[D_q|X = D]}.$$

From this we can further derive the same properties as the core model's semi-separating equilibrium with F^{*D} in place of F^{*}: $k_H^{*} > k_L^{*}$ if and only if $\frac{D_H}{D_L} > \frac{A_H}{A_L}$, i.e. $F < F^{*D}$. The price reaction to asset sales is positive (negative) if $k_H^{*} > (<) k_L^{*}$, and from this we can show that $k_H^{*} > (<) 0$ if and only if $F < (>) F^{*D}$.

We now demonstrate the robustness of the camouflage effect to allowing for debt issuance. As in Section 3, we give firms the option not to raise financing, and funds raised finance a new investment with expected return $r_q > 0$. The results are analogous to Propositions 3 and 4. The pooling and semi-separating equilibria considered above continue to hold if $1 + r_H \geq \frac{D_H}{D_L}$, because then all H-firms prefer raising financing to inaction. The camouflage effect is the analog of Proposition 4, part (iia). If $\frac{D_H}{D_L} > 1 + r_H > \frac{A_H}{A_L}(1 + \underline{k})$, then H-firms with $k > k_H^{*}$ forgo financing. They have no synergy motives to sell assets, as k is sufficiently high, and no investment motives to issue debt, because r_H is sufficiently low. Thus, debt is only issued by L-firms (with $k > k_L^{*}$); it offers no camouflage and is valued

[24] Note that there is a second effect of F in (15) which is absent in (2) and reinforces the balance sheet effect. When F rises, the amount that must be promised to debtholders P also rises. Since this is received in solvency regardless of firm quality, it is the same in the numerator and denominator and also reduces the LHS towards 1.

[25] One technical complication compared to the core model is that F^{*D} includes a conditional expectation that incorporates investor beliefs, so it will take on different values in different equilibria. In a debt-pooling equilibrium, the expectation evaluates to $\mathbb{E}[C + A]$, while in the asset-pooling equilibrium it evaluates to the strictly-smaller $C_L + A_L$. This means there is a gap between the sustainability regions of the two pooling equilibria, although a semi-separating equilibrium is still sustainable in this range if synergies are sufficiently strong.

at the lowest possible price of D_L. However, H-firms with sufficiently strong dissynergies ($k < k_H^*$, where $k_H^* > \underline{k}$) still sell assets. They thus offer camouflage and are valued at a pooled price of $\mathbb{E}[A|X = A] > A_L$. The threshold k_L^* is thus defined by $\frac{A_L(1+k_L^*)}{\mathbb{E}[A|X=A]} = 1$, which yields $k_L^* > 0$. As in Proposition 4, part (iia), L-firms exhibit a strict preference for asset sales. Even those with mildly positive synergies will sell assets, despite the loss of synergies, since doing so allows them to camouflage with H-firms.

We finally demonstrate the robustness of the correlation effect of Proposition 2 by turning to the negative correlation model. Type $(L, \overline{k})$ has the strongest incentive to issue debt, and so under an asset-pooling equilibrium, a deviator to debt is inferred as this type. Under this off-equilibrium belief, an asset-pooling equilibrium is sustainable if $\omega \geq \omega^{APE}$, where ω^{APE} is as in Proposition 2. On the other hand, a debt-pooling equilibrium is never sustainable, for the same reason that an equity-pooling equilibrium was unsustainable in the core model. Type $(H, \underline{k})$ has the strongest incentive to deviate to asset sales, so D1 requires an asset seller to be inferred as this type. Under this inference, $(H, \underline{k})$ will indeed deviate to asset sales. Thus, the correlation effect continues to hold.

C Negative Correlation, Additional Equilibria

C.1 Separating Equilibrium

No separating equilibrium was possible in the positive correlation model of Section 2.1. This subsection demonstrates that a separating equilibrium can hold in the negative correlation model of Section 2.2, if stock price concerns ω are sufficiently weak.

The equilibrium entails H selling assets and L issuing equity. Both types are fully revealed, so both fundamental value and the stock price, and thus the firm's objective function, equal $C_q + A_q$. If type H deviates to equity issuance, its payoff is

$$\omega(C_L + A_L) + (1 - \omega) \left(C_H + A_H + F - F \left(\frac{C_H + A_H + F}{C_L + A_L + F} \right) \right)$$

which is less than $C_H + A_H$ because $C_H + A_H > C_L + A_L$.

If type L deviates to asset sales, its payoff is

$$\omega(C_H + A_H) + (1 - \omega) \left(C_L + A_L + F - F \left(\frac{A_L}{A_H} \right) \right)$$

which is weakly less than $C_L + A_L$ if:

$$\omega \leq \frac{F \left(\frac{A_L}{A_H} - 1 \right)}{C_H - C_L + A_H - A_L + F \left(\frac{A_L}{A_H} - 1 \right)}.$$

Intuitively, if L deviates to asset sales, it suffers a capital loss on high-quality assets, but enjoys a stock price gain from being inferred as H. Thus, stock price concerns must be sufficiently low to deter deviation.

The equilibrium is formally stated in Proposition 5.

Proposition 5. *(Negative correlation, separating equilibrium). A separating equilibrium is sustainable in which type H sells assets and type L issues equity if* $\omega \leq \frac{F\left(\frac{A_L}{A_H}-1\right)}{C_H-C_L+A_H-A_L+F\left(\frac{A_L}{A_H}-1\right)}$.

C.2 Pooling Equilibria, General Model

This section shows that Proposition 2, the main result of Section 2.2, continues to hold in the general model which gives firms the choice of whether to raise capital, allows the capital raised to finance a positive-NPV investment, and introduces synergies. We impose two natural technical conditions that prevent synergies and investment returns, respectively, from dominating the other forces in the model. The first is given by

$$\mathbb{E}\left[k\right] < \frac{\pi}{F}\left(E_H - E_L\right). \tag{16}$$

Condition (16) ensures that, if a deviating firm is revealed as being low quality, it suffers a lower stock price. While this might seem automatic, deviation could technically increase the stock price if expected synergies are so large that, by deviating to equity sales, the market's expectation of saved synergies exceeds the inferred fall in firm quality and so gives the firm a higher stock price. A sufficient condition is $\mathbb{E}[k] = 0$, i.e. symmetrically-distributed synergies. In other words, (16) ensures that the asymmetry between positive and negative synergies is not so great as to swamp all other forces in the model and mean that a firm can increase its stock price by revealing itself as low-quality.

The second technical condition is given by

$$r_L < r_H + \frac{(A_L - A_H)(1+\overline{k})}{\mathbb{E}[A]}. \tag{17}$$

Intuitively, the presence of investment returns r discourages both H and L to deviate to inaction. If r_L is much greater than r_H, H could have the stronger incentive to deviate to inaction, even though L suffers the larger capital loss by pooling. Condition (17) ensures that this is not the case, so that asymmetry in investment returns does not swamp the other forces in the model. A sufficient condition is $r_L \leq r_H$, i.e. high-quality firms do not have lower-quality investment opportunities.

Our first result is that equity-pooling is never sustainable. D1 requires any deviator to be inferred as $(H, \underline{k})$: since $E_H > E_L$, this type makes the biggest capital loss by pooling on equity, and also has the strongest synergy motive to sell assets. Given this inference, $(H, \underline{k})$

will deviate to asset sales. By doing so, he sells assets for a fair value instead of issuing equity at a capital loss, and his stock price increases from $\mathbb{E}[C+A+Fr]$ to $C_H+A_H+F(r_H-\underline{k})$.[26]

Our second result is that an asset-pooling equilibrium is sustainable if ω is sufficiently high. D1 requires any deviator to be inferred as $(L,\overline{k})$, since its assets have the highest common-value and private-value components, and so it has the strongest incentive to deviate. Given this inference, $(L,\overline{k})$ would enjoy a fundamental gain by deviating, so the equilibrium requires both that deviation lowers the firm's stock price ($\mathbb{E}[C+A+F(r-k)] > C_L+A_L+Fr_L$, which holds due to condition (16)) and the stock price motive incentives ω to be sufficiently high. This requires

$$\omega \geq \frac{F\left(\frac{A_L(1+\overline{k})}{\mathbb{E}[A]}-1\right)}{\pi((C_H-C_L)-(A_L-A_H))+F\times\mathbb{E}[r-k]+F\left(\frac{A_L(1+\overline{k})}{\mathbb{E}[A]}-(1+r_L)\right)} \tag{18}$$

As in Proposition 2, asset-pooling requires ω to be sufficiently high so that the stock price decline deters $(L,\overline{k})$ from deviating from high-quality assets to low-quality equity.

Finally, we must show that neither type chooses to deviate to inaction. D1 requires any deviator to be inferred as quality L (given condition (17) and synergy $\overline{k}$ (since it has greatest incentive to retain its assets).[27] Given this inference, $(L,\overline{k})$ will deviate unless

$$\omega \geq \frac{F\left(\frac{A_L(1+\overline{k})}{\mathbb{E}[A]}-(1+r_L)\right)}{\pi((C_H-C_L)-(A_L-A_H))+F\times\mathbb{E}[r-k]+F\left(\frac{A_L(1+\overline{k})}{\mathbb{E}[A]}-(1+r_L)\right)}. \tag{19}$$

Since $r_L \geq 0$, this is a looser bound than in (18) and so can be ignored.

These results are summarized in Proposition 6, the analog of Proposition 2:

Proposition 6. *(Negative correlation, pooling equilibria, voluntary capital raising.) Assume conditions (16) and (17). An equity-pooling equilibrium is never sustainable. An asset-pooling equilibrium is sustainable if and only if*

$$\omega \geq \frac{F\left(\frac{A_L(1+\overline{k})}{\mathbb{E}[A]}-1\right)}{\pi((C_H-C_L)-(A_L-A_H))+F\times\mathbb{E}[r-k]+F\left(\frac{A_L(1+\overline{k})}{\mathbb{E}[A]}-(1+r_L)\right)}. \tag{20}$$

In this equilibrium, all firms sell assets for $\mathbb{E}[A]=\pi A_H+(1-\pi)A_L$. *If equity is sold (off-equilibrium), it is inferred as type L and valued at* E_L. *The stock prices of asset sellers and equity issuers are* $\mathbb{E}[C+A+F(r-k)]$ *and* $C_L+A_L+Fr_L$, *respectively.*

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[26] Recall that $\underline{k} < 0 \leq r_H$. In addition, differing from Section 2.2, the stock price here incorporates both expected investment returns and, in the case of asset sales, expected synergy losses.

[27] Note that there was no need to deal with off-equilibrium beliefs about a firm deviating to inaction in Section 3.1, because $\omega = 0$ meant that inactive firms were unconcerned with the stock market's inferences from inaction.

D Selling the Core Asset

D.1 Positive Correlation

This subsection extends the core positive correlation model of Section 2.1 to allow the firm to sell the core asset (in addition to the non-core asset and equity). Proposition 7 below characterizes which equilibria are sustainable and when.

Proposition 7. *(Positive correlation, selling the core asset.) Consider a pooling equilibrium where all firms sell non-core assets ($X = A$) and a firm that sells equity or the core asset is inferred as L. The prices of core assets, non-core assets, and equity are C_L, $\pi A_H + (1-\pi)A_L$, and $C_L + A_L + F$, respectively. This equilibrium is sustainable if:*

$$F \leq F^* \equiv \frac{C_H A_L - C_L A_H}{A_H - A_L} \text{ and} \tag{21}$$

$$\frac{A_H}{A_L} \leq \frac{C_H}{C_L}. \tag{22}$$

Consider a pooling equilibrium where all firms sell core assets ($X = C$) and a firm that sells equity or the non-core asset is inferred as L. The prices of core assets, non-core assets, and equity are $\pi C_H + (1-\pi)C_L$, A_L, and $C_L + A_L + F$, respectively. This equilibrium is sustainable if:

$$F \leq F^{*C} \equiv \frac{C_L A_H - C_H A_L}{C_H - C_L} \text{ and} \tag{23}$$

$$\frac{A_H}{A_L} \geq \frac{C_H}{C_L}. \tag{24}$$

Consider a pooling equilibrium where all firms sell equity ($X = E$) and a firm that sells either asset is inferred as L. The prices of core assets, non-core assets, and equity are C_L, A_L, and $\pi(C_H + A_H) + (1-\pi)(C_L + A_L) + F$, respectively. This equilibrium is sustainable if:

$$F \geq F^* \equiv \frac{C_H A_L - C_L A_H}{A_H - A_L} \text{ and} \tag{25}$$

$$F \geq F^{*C} \equiv \frac{C_L A_H - C_H A_L}{C_H - C_L}. \tag{26}$$

For the asset-pooling equilibrium, condition (21) is the same as condition (ib) in Proposition 1 of the core model: it ensures that the only admissible off-equilibrium belief under D1 is that an equity issuer is of quality L. Equation (22) is new and similarly ensures that the belief that a core asset seller is of quality L satisfies D1. Thus, the asset-pooling equilibrium can only be sustained if non-core assets have less information asymmetry than core assets, as is intuitive. For the core-asset-pooling equilibrium, equations (23) and (24) similarly guarantee that the off-equilibrium belief that a seller of the non-core asset or equity is of quality L satisfies D1. To understand the intuition, note that $F \lesseqgtr F^*$ can be rewritten as $\frac{A_H}{A_L} \lesseqgtr \frac{E_H}{E_L}$, while $F \lesseqgtr F^{*C}$ can similarly be rewritten $\frac{C_H}{C_L} \lesseqgtr \frac{E_H}{E_L}$.

The main result of Proposition 7 is to show that an equity-pooling equilibrium is still sustainable. Condition (25) is the same as condition (iib) of Proposition 1 in the core model: it means that the off-equilibrium belief that a seller of the non-core asset is of quality L satisfies D1. Equation (26) is new and guarantees that the belief that a core-asset seller is of quality L also satisfies D1. If both inequalities are satisfied, equity issuance is sustainable even though it does not exhibit the least information asymmetry (absent the balance sheet effect). One of the assets (core or non-core) will exhibit more information asymmetry than the other; since equity is a mix of both assets, its information asymmetry will lie in between. Even though equity is never the safest claim, it may still be issued if F is sufficiently large, due to the balance sheet effect.

D.2 Negative Correlation

We now move to the negative correlation case. Proposition 8 characterizes the pooling equilibria.

Proposition 8. *(Negative correlation, selling the core asset.) The only sustainable pooling equilibrium is one in which all firms sell non-core assets ($X = A$) and a firm that sells equity or the core asset is inferred as L. The prices of core assets, non-core assets, and equity are C_L, $\pi A_H + (1-\pi)A_L$, and $C_L + A_L + F$, respectively. This equilibrium is sustainable if:*

$$\omega \geq \frac{F\left(\frac{A_L}{E[A]} - 1\right)}{\pi((C_H - C_L) - (A_L - A_H)) + F\left(\frac{A_L}{E[A]} - 1\right)}. \tag{27}$$

An equity-pooling equilibrium is not sustainable by the same logic as in the main text. The same intuition rules out a core-asset-pooling equilibrium since, like equity, the core asset is positively correlated with firm quality. In either equilibrium, the only off-equilibrium belief consistent with D1 is that a deviator to non-core assets is of quality H. Under this belief, H will indeed deviate to non-core assets. Thus, even if the core asset exhibits less information asymmetry than the non-core asset, and so MM would suggest that it is more likely to be sold, a core-asset-pooling equilibrium cannot exist due to the correlation effect.

Equation (27) is the condition for L not to deviate to equity, and is the same as equation (3) in the core model. If L deviates to the core asset, his objective function is also $C_L + A_L$ and so we have the same condition. This is intuitive: regardless of whether it deviates to the core asset or equity, its claim is fairly priced as it is revealed as L. The off-equilibrium belief that a seller of the core asset or equity is of quality L is trivially consistent with D1: For either claim, L has a stronger incentive than H to issue it regardless of the off-equilibrium belief, because it will retain valuable non-core assets while selling less-valuable core assets or equity.

The semi-separating equilibria are very similar to the core model. There is a semi-separating equilibrium where H sells non-core assets and L issues equity, and also a semi-separating equilibrium where H sells non-core assets and L sells core assets, under exactly the same conditions as in the core model. In both equilibria, by deviating, L's stock price increases but his fundamental value falls by $\frac{F(A_L - A_H)}{A_H}$. Regardless of whether L sells equity or core assets in the semi-separating equilibrium, deviation involves him selling his highly-valued non-core assets and thus suffering a loss. In both cases, the off-equilibrium belief that a deviator to the off-equilibrium claim is of quality L is consistent with D1, because L has strictly stronger incentives than H to do so. There is no semi-separating equilibrium where H sells core assets and L sells equity, or when H sells equity or L sells the core asset, since L will mimic H in both cases.

E Financing from Multiple Sources

The core model assumes that firms can only raise financing from a single source. One potential justification is that the transactions costs from using multiple sources of financing are prohibitive. This section instead allows firms to choose a combination of financing sources, and shows that the equilibria of the core model continue to hold if a firm choosing multiple sources is inferred as L; we further show that this off-equilibrium belief is consistent with D1.

We start with positive correlation. The action space now consists of a fraction $\alpha \in [0,1]$ of financing that is raised from the pooling claim, which is no longer restricted to be 0 or 1. The off-equilibrium belief consists of a potentially different value π_α for each choice of α. We use X_α to denote the valuation of claim X applying the belief π_α, i.e. $X_\alpha = \pi_\alpha X_H + (1-\pi_\alpha) X_L$. To sustain the pooling equilibria of the core model, we specify $\pi_\alpha = 0$ for all $\alpha \in (0,1)$, i.e. any firm using both sources is believed to be type-L. We show that this off-equilibrium belief is consistent with D1 and that the pooling equilibria of the core model continue to hold under this off-equilibrium belief.

To do so, we consider the incentive of a type q to deviate to an action $\alpha \in (0,1)$. Given the resulting off-equilibrium belief, the type deviates if the unit cost of financing is lower,

$$\alpha \left(\frac{c(X,q)}{X_\alpha} \right) + (1-\alpha) \left(\frac{c(\tilde{X},q)}{\tilde{X}_\alpha} \right) < \frac{c(X,q)}{\mathbb{E}[X]}$$

Dividing both sides by $c(X,q)$, and invoking from Proposition 1 the condition $F < F^*$ for the asset-pooling equilibrium and $F > F^*$ for the equity-pooling equilibrium, we see that this deviation is most likely for type L, and so D1 requires $\pi_\alpha = 0$ for any $\alpha \neq 1$. This is sufficient to verify the sustainability of the equilibrium: Type q will not deviate, given the above off-equilibrium belief, if

$$\alpha\left(\frac{c(X,q)}{X_L}\right)+(1-\alpha)\left(\frac{c(\tilde{X},q)}{\tilde{X}_L}\right)>\frac{c(X,q)}{\mathbb{E}[X]}.$$

We have $\frac{c(X,q)}{\mathbb{E}[X]}<\frac{c(X,q)}{X_L}$, and the conditions in Proposition 1 (to guarantee incentive compatibility) yield $\frac{c(X,q)}{\mathbb{E}[X]}<\frac{c(\tilde{X},q)}{\tilde{X}_L}$. Thus, the linear combination of these inequalities continues to hold.

Moving to negative correlation, the equity-pooling equilibrium continues to be unsustainable by the same logic as in Proposition 2, since we have expanded the action space compared to the core model. Turning to the asset-pooling equilibrium, type q deviates to a given α if

$$\omega\Big[(\pi_\alpha-\pi)((C_H-C_L)-(A_L-A_H))\Big]$$
$$>(1-\omega)F\left(\alpha\frac{A_q}{A_\alpha}+(1-\alpha)\frac{E_q}{E_\alpha}-\frac{A_q}{\mathbb{E}[A]}\right)$$

We first show that $\pi_\alpha=0$ is consistent with D1 for any α, although we can no longer prove that this is the *only* belief consistent with D1. We seek a belief π_α under which L has the strongest incentive to deviate. If $\pi_\alpha=0$, the inequality simplifies to $\frac{E_q}{\mathbb{E}[E]}<\frac{A_q}{\mathbb{E}[A]}$, which does not depend on α. Under this belief, type L has at least as strong an incentive to deviate as any other type, so this belief is consistent with D1.

F Correlation Effect With General Correlation

While the core model allows for perfect positive correlation ($A_H>A_L$) and perfect negative correlation ($A_H<A_L$), this section considers a generalization that allows for any degree of correlation between the core and non-core assets. It shows that the correlation effect of Section 2.2 does not require perfect negative correlation; indeed, it can continue to hold even if the correlation is positive.

In this generalized model, let $C_H>C_L$ and $A_H>A_L$ throughout, where we assume $C_H+A_L>C_L+A_H$ so that a high-quality firm has a higher total asset value. Let π be the probability that core assets are of quality H and $\rho\equiv\Pr(A_H|C_H)=\Pr(A_L|C_L)$ denote the correlation between the values of the core and non-core assets. Thus, $(C,A)=(C_H,A_H)$ with probability ("w.p.") $\pi\rho$, (C_H,A_L) w.p. $\pi(1-\rho)$, (C_L,A_H) w.p. $(1-\pi)(1-\rho)$, and (C_L,A_L) w.p. $(1-\pi)\rho$. The perfect positive (negative) correlation model of Section 2.1 (2.2) corresponds to $\rho=1$ ($\rho=0$), i.e. high-quality core assets always coincide with high-quality (low-quality) non-core assets. (Indeed, in Section 2.2, we relabeled A_H and

A_L to emphasize this point.) When $\rho = \frac{1}{2}$ we have $\Pr(A_H|C_H) = \Pr(A_H|C_L)$, i.e. zero correlation. Thus, $\rho > (<) \frac{1}{2}$ corresponds to general, i.e. not necessarily perfect, positive (negative) correlation.

Proposition 9 is the analogy of Proposition 2 in the core model. It states that, when the correlation ρ is below a cutoff ρ^* (which may be strictly greater than $\frac{1}{2}$), an equity-pooling equilibrium is not sustainable for any ω but an asset-pooling equilibrium may be sustainable for sufficiently high ω. Proposition 2 was a special case of this result with $\rho = 0$.

Proposition 9. *(Correlation effect, general correlation.) An equity-pooling equilibrium is not sustainable if and only if*

$$\rho < \rho^* \equiv \begin{cases} \left(\frac{1-\pi}{2\pi-1}\right)\left(\frac{(C_H-C_L)-(A_H-A_L)}{A_H-A_L}\right) & \text{if } \pi > \frac{1}{2}, \\ \infty \text{ if } \pi \leq \frac{1}{2}, \end{cases} \tag{28}$$

An asset-pooling equilibrium is sustainable if and only if

$$\omega \geq \omega^{APE} \equiv \frac{F\left(\frac{A_H}{\mathbb{E}[A]} - 1\right)}{\mathbb{E}[C+A] - (C_L + A_H) + F\left(\frac{A_H}{\mathbb{E}[A]} - 1\right)} > 0. \tag{29}$$

In this equilibrium, all firms sell assets for $\mathbb{E}[A] = [\pi\rho + (1-\pi)(1-\rho)]A_H + [\pi(1-\rho) + \rho(1-\pi)]A_L$ *and are priced at* $\mathbb{E}[C+A]$. *If equity is sold (off-equilibrium), it is inferred as stemming from* (C_L, A_H) *and valued at* $C_L + A_H$.

We first discuss the asset-pooling equilibrium, and start by addressing off-equilibrium beliefs. Given an arbitrary off-equilibrium belief $(\tilde{C}, \tilde{A})$, a type (C, A) deviates to equity if and only if $\frac{C+A+F}{\tilde{C}+\tilde{A}+F} - \frac{A}{\mathbb{E}[A]} < \kappa$, where $\kappa \equiv \left(\frac{\omega}{1-\omega}\right)\left(\frac{1}{F}\right)\left(\tilde{C} + \tilde{A} - \mathbb{E}[C+A]\right)$ is independent of firm type. The inequality is most easily satisfied for $C = C_L$, so D1 requires that a deviator be inferred as having this value. D1 does not uniquely pin down a valuation for A with general ω, but to be consistent with the $\omega = 0$ model, we set $\tilde{A} = A_H$.

Given this off-equilibrium belief, consider the incentives of (C_L, A_H), the type most likely to deviate. Deviating to equity issuance allows it to break even, compared to a capital loss from pooling on asset sales), while causing the stock price to fall from $\mathbb{E}[C+A]$ to $C_L + A_H$. Thus, the stock price decline outweighs the fundamental value gain, sustaining the asset-pooling equilibrium, if and only if stock price concerns are high enough to satisfy (29).

The intuition is as in the core model of Section 2.2: Regardless of the correlation between the core and non-core assets ρ, the equity sold is perfectly positively correlated with the rest of the firm, and so deviation leads to a low stock price for both. Indeed, when $\rho = 0$, the bound ω^{APE} simplifies to the same value as in Proposition 2 of that section (in which A_H and A_L were relabeled).

We now turn to the equity-pooling equilibrium, and again start by addressing off-equilibrium beliefs. Given an arbitrary off-equilibrium belief $(\tilde{C}, \tilde{A})$, a type (C, A) deviates to asset sales if and only if $\frac{A}{\tilde{A}} - \frac{C+A+F}{\mathbb{E}[C+A]+F} < \kappa'$, where $\kappa' \equiv \left(\frac{\omega}{1-\omega}\right)\left(\frac{1}{F}\right)\left(\tilde{C} + \tilde{A} - \mathbb{E}[C+A]\right)$ is independent of type. D1 immediately requires $\tilde{C} = C_H$, but does not uniquely pin down $\tilde{A}$. To be consistent with the $\omega = 0$ model, we set $\tilde{A} = A_L$.

Now consider the incentive of type (C_H, A_L) to deviate to asset sales. If $\rho < \rho^*$, then $C_H + A_L > \mathbb{E}[C+A]$, so both the stock price and fundamental value are higher for (C_H, A_L) if he deviates to asset sales. This means the equilibrium is never sustainable, regardless of ω. If $\rho \geq \rho^*$, then $C_H + A_L < \mathbb{E}[C+A]$. In this case, both the stock price and the fundamental value are higher for (C_H, A_L) by cooperating with equity issuance than by deviating, so the equilibrium is always sustainable, again regardless of ω.

For relatively low ρ, deviation leads to negative inferences about the quality of non-core assets. This is because the type most likely to deviate has low-quality non-core assets (thus wishing to sell assets) and high-quality core assets (thus not wishing to sell equity). Even if the correlation between core and non-core assets is not perfectly negative, deviation leads to negatively-correlated *inferences* on the values of core and non-core assets. Indeed, $\rho^* > \frac{1}{2}$ if and only if $\pi < 1 - \frac{1}{2}\left(\frac{A_H - A_L}{C_H - C_L}\right)$. The correlation could be positive and yet the correlation effect still holds, again because deviation leads to negatively-correlated inferences.

While ρ does not affect the inference on the deviating firm, its role is instead to affect the pooled stock price (and thus whether deviating improves or worsens the inference on the rest of the firm compared to pooling) and pooled values of assets and equity (and thus whether deviating improves or worsens the capital gain compared to pooling). When ρ is low, type (C_H, A_H) is rare – it is unlikely that both core and non-core assets are high-quality. Thus, the pooled stock price $\mathbb{E}[C+A]$, which incorporates the possibility that the firm is of type (C_H, A_H), is lower.[28] In this case, deviation improves both the stock price and fundamental value, and so the equity-pooling equilibrium is unsustainable. For the same reason, low ρ means that $C_H + A_L > \mathbb{E}[C+A]$ and so type (C_H, A_L) was making a strictly positive fundamental loss of $\frac{C_H+A_L+F}{\mathbb{E}[C+A]+F} - 1$ under equity issuance, thus giving it fundamental value motives to deviate.

A similar effect can occur for the asset-pooling equilibrium. If $\pi < \frac{1}{2}$ and ρ is sufficiently small, we could have $\mathbb{E}[C+A] < C_L + A_H$. In this case (C_L, A_H) would enjoy a higher stock price and higher fundamental value from deviating to equity issuance, and so asset pooling never be sustainable.[29] However, $\mathbb{E}[C+A] < C_L + A_H$ is a stronger condition than $\mathbb{E}[C+A] < C_H + A_L$ (since $C_L + A_H < C_H + A_L$). Thus, the values of (π, ρ) for which

[28] (C_L, A_L) also becomes rarer if ρ falls, but the effect of this on the pooled stock price is smaller if $\pi \geq \frac{1}{2}$, as C_L is rarer than C_H to begin with. If on the other hand $\pi < \frac{1}{2}$, then $E[C+A] < C_H + A_L$ and so it is automatic that, regardless of ρ, (C_H, A_L) enjoys an increase in both fundamental value and the stock price from deviation and so an equity-pooling equilibrium is unsustainable.

[29] This is already accounted for in the statement of Proposition 9, as the value of ω^{APE} exceeds 1 in this case.

equity-pooling is sustainable is a strict subset of those for which asset-pooling is sustainable.

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